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Mathematical modelling of swirling flow in hydraulic turbines for the full operating range

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ABSTRACT

We introduce and validate a novel mathematical model for computing the radial profiles of both axial and circumferential velocity components, respectively, of the swirling flow exiting the runner of hydraulic turbines within the full operating range. We assume an incompressible, inviscid, axisymmetrical, and steady swirling flow, with vanishing radial velocity at runner outlet. First we find the correlation between the flux of moment of momentum downstream the turbine runner and the operating regime given by turbine's discharge and head. Second, we express the relationship between the axial and circumferential velocity components, corresponding to the fixed pitch runner blades, using the swirl-free velocity instead of the traditional relative flow angle at runner outlet. It is shown that the swirl-free velocity profile practically does not change with the operating regime. Third, we introduce a constrained variational problem corresponding to the minimization of the flow force while maintaining the prescribed discharge and flux of moment of momentum. This formulation also accounts for a possible central stagnant region to develop when operating the turbine far from the best efficiency point. Fourth, we show that by representing the unknown axial velocity profile with a suitable Fourier–Bessel series, the discharge constraint can be automatically satisfied. The resulting numerical algorithm is robust and produces results in good agreement with available data for both axial and circumferential velocity profiles measured on a model Francis turbine at several operating regimes. Our mathematical model is suitable for the early optimization stages of the runner design, as it provides the swirling flow configuration at runner outlet without actually computing the runner. By optimizing the parameterized swirl-free velocity profile one can achieve through the inverse design approaches the most suitable runner blades configuration at the trailing edge.

1. Introduction

Modern hydraulic turbines meet new challenges associated with the variable demand on the energy market as well as limited energy storage capabilities, resulting in great flexibility required in operation. Quite often turbines tend to be operated over an extended range of regimes far from the best efficiency point. In particular, Francis turbines, which have a fixed pitch runner, experience an abrupt decrease in efficiency and severe pressure fluctuations at off-design operating regimes. In

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(54) **LIQUID CONTROL JET DURING PART LOAD OPERATION IN A HYDRAULIC TURBINE**

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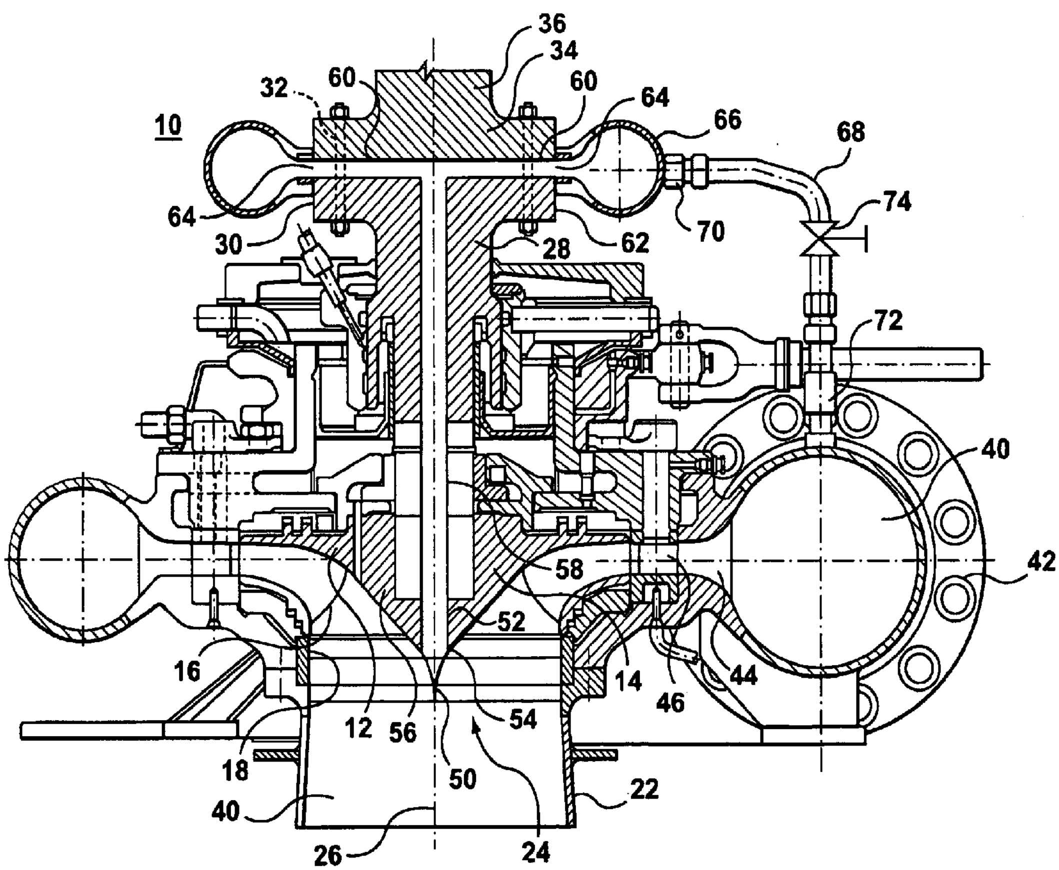
(57) **ABSTRACT**

A hydraulic turbine including: a passageway permitting liquid to pass through the turbine; a draft tube defining a portion of the passageway through which liquid normally flows in a vortex flow path during optimal turbine operating conditions; a rotatable runner mounted upstream of the draft tube and rotating about a central axis passing through the runner and extending into the draft tube; at least one nozzle head device positioned relative to the central axis of the runner and adjacent to an upper portion of the draft tube, the at least one nozzle head device has at least one nozzle from which a corresponding control jet of high velocity liquid is injected axially downstream of the runner and into liquid flowing into the upper portion of the draft tube during part load turbine operation, so as to mitigate breakdown of the vortex flow path.



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Analysis and Prevention of Vortex Breakdown in the Simplified Discharge Cone of a Francis Turbine

We perform a numerical analysis of the decelerated swirling flow into the discharge cone of a model Francis turbine operated at variable discharge and constant head, using an axisymmetric turbulent swirling flow model and a corresponding simplified computational domain. Inlet boundary conditions correspond to velocity and turbulent kinetic energy profiles measured downstream the Francis runner. Our numerical results are validated against experimental data on a survey section further downstream in the cone, showing that the Reynolds stress turbulence model with a quadratic pressure-strain term correctly captures the flow field. It is shown that the diffuser performance quickly deteriorates as the turbine discharge decreases, due to the occurrence and development of vortex breakdown, with a central quasistagnant region. We investigate a novel flow control technique, which uses a water jet injected from the runner crown tip along the axis. It is shown that the jet discharge can be optimized for minimum overall losses, while the vortex breakdown is eliminated. This flow control method is useful for mitigating the Francis turbine flow instabilities when operating at partial discharge.
[DOI: 10.1115/1.4001486]

1 Introduction

The variable demand on the energy market, as well as the limited energy storage capabilities, requires a great flexibility in operating hydraulic turbines. As a result, turbines tend to be operated over an extended range of regimes quite far from the best efficiency point. In particular, Francis turbines operated at partial discharge have a high level of residual swirl at the draft tube inlet as a result of the mismatch between the swirl generated by the guide vanes and the angular momentum extracted by the turbine runner [1]. Further downstream, the decelerated swirling flow in the draft tube cone often results in vortex breakdown, which is recognized now as the main cause of severe flow instabilities and pressure fluctuations experienced by hydraulic turbines operated at part load [2]. More than 3 decades ago Palde [3] concluded that the draft tube surge is a hydrodynamic instability, known as vortex breakdown, occurring in the draft tube as a result of rotation remaining in the fluid as it leaves the turbine runner and enters the draft tube throat.

The main goal of a hydraulic turbine draft tube is to decelerate the flow exiting the runner, thereby converting the excess of kinetic head into static head. Modern hydraulic turbines have compact elbow draft tubes, with a rather short discharge cone. As a result, the draft tube hydrodynamics is very complex due to the combination of swirling flow deceleration with flow direction and cross-section shape/area changes. In practice, the hydraulic performance of a turbine draft tube is quantified with the wall pressure recovery coefficient

$$\chi \equiv \frac{\left(\frac{p_{\text{wall}}}{\rho} + gz\right)_{\text{outlet}} - \left(\frac{p_{\text{wall}}}{\rho} + gz\right)_{\text{inlet}}}{\frac{Q^2}{2A_{\text{inlet}}^2}\left[1 - \left(\frac{A_{\text{inlet}}}{A_{\text{outlet}}}\right)^2\right]} \simeq \frac{(p_{\text{wall}}/\rho + gz)_{\text{outlet}} - (p_{\text{wall}}/\rho + gz)_{\text{inlet}}}{Q^2/(2A_{\text{inlet}}^2)} \quad (1)$$

Most of the pressure recovery occurs in the draft tube cone. For economical reasons, this conical diffuser is short and it has a rather large angle. To minimize hydraulic losses associated with kinetic-to-potential energy conversion in the draft tube cone, a certain level of residual swirl is provided at the runner outlet. This swirling flow at the cone inlet is tuned for optimal performance at

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Analysis of the Swirling Flow Downstream a Francis Turbine Runner

An experimental and theoretical investigation of the flow at the outlet of a Francis turbine runner is carried out in order to elucidate the causes of a sudden drop in the draft tube pressure recovery coefficient at a discharge near the best efficiency operating point. Laser Doppler anemometry velocity measurements were performed for both axial and circumferential velocity components at the runner outlet. A suitable analytical representation of the swirling flow has been developed taking the discharge coefficient as independent variable. It is found that the investigated mean swirling flow can be accurately represented as a superposition of three distinct vortices. An eigenvalue analysis of the linearized equation for steady, axisymmetric, and inviscid swirling flow reveals that the swirl reaches a critical state precisely (within 1.3%) at the discharge where the sudden variation in draft tube pressure recovery is observed. This is very useful for turbine design and optimization, where a suitable runner geometry should avoid such critical swirl configuration within the normal operating range. [DOI: 10.1115/1.2137341]

1 Introduction

Swirling flow behavior in various technical applications has long been an intensive subject of research. Usually swirl effects are seen as either the desired result of design or unavoidable, possibly unforseen, side effects [1]. However, the hydraulic turbine draft tube on one hand benefits from the swirl at the runner outlet in order to mitigate flow detachment in the cone, but on the other hand suffers from the flow instabilities leading to pressure fluctuations and ultimately to the draft tube surge.

The draft tube of a hydraulic turbine is the machine component where the flow exiting the runner is decelerated, thereby converting the excess of kinetic energy into static pressure. In the case of machine rehabilitation of an existing power plant, mostly only the runner and the guide vanes are currently modified. For economical and safety reasons, the spiral casing and the draft tube are seldom redesigned, even if these components present some undesirable behavior. However, the installation of an upgraded runner requires a reliable prediction of the flow in a compact draft tube in order to avoid the peculiar and undesirable efficiency curve from Fig. 1. The efficiency drop as the discharge is increased above the best efficiency point value is found to be related to a corresponding sudden variation in the draft tube pressure recovery coefficient at the same discharge. It is this phenomenon we address in this paper.

The obvious practical importance of predicting the complex flow downstream the turbine runner, in the draft tube, led to the FLINDT research project of Flow Investigation in Draft Tubes [2]. The main objective of this project was to investigate the flow in hydraulic turbines draft tubes, for a better understanding of the physics of these flows and to build up an extensive experimental data base describing a wide range of operating points which can provide a firm basis for the assessment of the CFD engineering practice in this component. The extensive experimental investigation of the draft tube flow has been complemented with three-dimensional numerical flow simulations [3,4] aimed at elucidating the swirling flow evolution up to the turbine outlet as well as the phenomena that led to the peculiar sudden drop in the turbine efficiency.

Other investigations have been mainly focused on the ability of the CFD tools to accurately reproduce the complex three-dimensional velocity and pressure field in draft tubes for Kaplan turbines [5,6]. One important issue addressed in these studies was the sensitivity of numerical results to the boundary conditions, particularly the inlet ones.

The present paper focuses on the structure of the swirl produced by the constant pitch turbine runner and further ingested by the draft tube. The corresponding hydrodynamic field is a direct outcome of the runner design and the operating point. Since changing the runner design, while keeping the same draft tube, may lead to an unexpected sudden efficiency drop for a certain discharge, it would be preferable that some design criteria be put forward as far as the runner outlet swirl is concerned. The present analysis shapes such criteria by using relative simple mathematical and numerical tools. Of course, the complex three-dimensional and unsteady flow in the draft tube cannot be quantitatively predicted only by analyzing the draft tube inlet swirl. However, if the runner outlet swirl structure displays a sudden change with respect to appropriate criteria, and this change occurs at a discharge close to the experimental one where the sudden drop in turbine efficiency is observed, these criteria should be taken into account when designing or redesigning the runner.

In analyzing a swirling flow one benefits from a large body of literature on this subject. In laboratory investigations swirl was

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Domain-Decomposition Method for Time-Harmonic Aeroacoustic Problems

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A domain-decomposition (DD) method is developed for parallel computation of time-harmonic aerodynamic–aeroacoustic problems. The computational domain is decomposed into subdomains, and the aerodynamic–aeroacoustic boundary-value problem is solved independently for each subdomain. Impedance-type transmission conditions are imposed on the artificially introduced subdomain boundaries to ensure the uniqueness of the solution. A Dirichlet-to-Neumann map is used as a nonreflecting radiation condition along the outer computational boundary. Subdomain problems are then solved using the finite element method, and an iterative scheme updates the transmission conditions to recover the global solution. The present algorithm is implemented for two model problems. First, the sound radiated from a surface simulating a two-dimensional monopole is calculated using an unstructured mesh. Second, the flow about a thin airfoil in a transverse gust is computed using a structured mesh. The accuracy of the numerical scheme is validated by comparison with existing solutions for both the near-field unsteady pressure and the far-field radiated sound. The convergence and the computational time and memory requirements of the present method are studied. It is shown that by combining the subdomain direct solvers with global iterations this DD method significantly reduces both the computational time and memory requirements.

I. Introduction

ACCURATE computation of aeroacoustics phenomena must preserve the wave form with minimum dispersion and dissipation. This requires a large number of grid points per wavelength throughout the computational domain. As a result, for time-harmonic waves the discrete form of the corresponding elliptic boundary-value problem usually leads to a very large system of equations.

There are two types of solvers for large system of equations, direct solvers and iterative solvers. Direct solvers are robust, but for aeroacoustic applications their memory and CPU time requirements limit their use in three-dimensional and/or high-frequency computations. Iterative solvers, on the other hand, have less memory requirements; however, their convergence rate strongly depends on the wave number, and in some instances they may not converge at all.

This paper presents a method for solving time-harmonic aerodynamic and aeroacoustic problems using domain decomposition (DD). The method has been developed for the exterior Helmholtz problem by Susan-Resiga and Atassi.[1] The computational domain is divided into subdomains, and the boundary-value problem is formulated and solved independently for each subdomain. On the artificially introduced subdomain boundaries, transmission conditions that ensure the uniqueness of the solution are imposed. An iterative scheme updates the transmission conditions after every iteration until the global solution is recovered. Thus, the problem can be solved concurrently in all subdomains on parallel computers. The main advantage of the present method is to reduce memory requirements for solving large systems of linear equations. Moreover, with parallel computing the computational time is also significantly reduced. The method thus provides an efficient and robust scheme for solving large systems of linear algebraic equations.

In the present paper the DD method is implemented for two model problems. First, sound radiated from a surface simulating a two-dimensional monopole is calculated using the finite element method on unstructured mesh. This model problem is an extension to that developed in Ref. 1 for a structured mesh. A nonoverlapping mesh partitioning is used to build an overlapping domain decomposition. The algorithm also defines the new subdomain boundaries, other than the original boundaries of the global computational domain, on which transmission conditions are imposed.

Second, the unsteady flow about a thin airfoil in a transverse gust is computed to determine both the near-field unsteady pressure along the airfoil surface and the far-field radiated sound. The problem is formulated in terms of the unsteady linearized Euler equations, with the unsteady pressure as the unknown variable. This has the advantage of avoiding the velocity discontinuity in the wake and leads to a simple formulation for the radiation condition on the outer computational boundary. It also tests the accuracy of the present DD method for treating singularities such as the airfoil leading-edge singularity.

The present method uses an exact nonreflecting radiation condition based on a Dirichlet-to-Neumann map. The support of the kernel of the map is truncated to enhance the efficiency of the computations. The finite element (FE) method is used to discretize the boundary-value problem in each case. The flux associated with each subdomain interface node is used for implementing the transmission conditions. The accuracy of the numerical scheme has been validated by comparison with the exact analytical solution for the monopole problem and with the results of Atassi et al.[2] for both the near-field unsteady pressure along the airfoil surface and the far-field acoustic pressure.

Finally, the performance of the DD method is examined. The dependence of the convergence rate on the domain overlap and on the problem parameters (the Mach number and the reduced frequency) is investigated. The benefits of using DD for computational time and memory requirements are compared with a direct solver for the whole computational domain. It is also shown that, because of the limited communication between subdomains, the DD algorithm is particularly suitable for parallel computing.

II. Formulation

The Helmholtz equation is the basic equation for time-harmonic waves. It also represents, through a transformation, the linearized Euler equations about a uniform mean flow.[3] We therefore consider the Helmholtz equation for the development and application of our DD approach.

Let us consider an infinite domain exterior to an inner boundary Γ, as shown in Fig. 1a. The outward unit vector normal to Γ is denoted

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A Domain Decomposition Method for the Exterior Helmholtz Problem

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A new domain decomposition method is presented for the exterior Helmholtz problem. The nonlocal Dirichlet-to-Neumann (DtN) map is used as a nonreflecting condition on the outer computational boundary. The computational domain is divided into nonoverlapping subdomains with Sommerfeld-type conditions on the adjacent subdomain boundaries to ensure uniqueness. An iterative scheme is developed, where independent subdomain boundary-value problems are obtained by applying the DtN operator to values from the previous iteration. The independent problems are then discretized with finite elements and can be solved concurrently. Numerical results are presented for a two-dimensional model problem, and both the solution accuracy and convergence rate are investigated. © 1998 Academic Press

Key Words: Helmholtz equation; exterior problem; domain decomposition; nonreflecting boundary conditions; finite element method.

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1. INTRODUCTION

In acoustics, the Helmholtz equation is the basic equation governing the propagation and scattering of time-harmonic sound. As a result, it has been extensively treated both analytically and numerically. However, as the field of applications extends to more complex problems and geometries, one needs to develop more accurate and powerful computational methods. Computational acoustics must resolve the waveform with minimum dispersion and dissipation. This requirement leads to a large number of grid points and, as a consequence, one ends up solving a very large system of equations. Numerical methods developed for solving such systems generally use iterative schemes. Because of the large memory requirements and to take advantage of parallel implementation, more recent iterative schemes are based on Schwarz domain decomposition wherein the computational domain is divided into several smaller subdomains which can be treated concurrently.

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A Domain Decomposition Method for the Exterior Helmholtz Problem

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1. INTRODUCTION

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