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# Mixed-mode Testing for an Asymmetric Four-point Bending Configuration of Polyurethane Foams

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**Abstract.** Many efforts have been made recently to determine the fracture toughness of different types of foams in static and dynamic loading conditions. Taking into account that there is no standard method for the experimental determination of the fracture toughness of plastic foams, different procedures and specimens were used. This paper presents the polyurethane foam fracture toughness results obtained experimentally for three foam densities. Asymmetric four-point bending specimens were used for determining fracture toughness in mode I and in a mixed one, and also the influence of the loading speed and geometry of the specimen were investigated.

## Introduction

Polyurethane (PUR) foam materials are widely used as cores in sandwich composites, for packing and cushioning. They are made of interconnected networks of solid struts and cell walls incorporating voids with entrapped gas. The main characteristics of foams are lightweight, high porosity and good energy absorption capacity, [1, 2]. Foam materials crush in compression, while in tension fail by propagating of single crack, [3]. Most of the rigid polymeric foams have a linear – elastic behavior in tension up to fracture, and a brittle failure behavior. So, they can be treated using fracture criteria of Linear Elastic Fracture Mechanics (LEFM).

Consequently, the fracture toughness of such foams became an important characteristic, because cracks weakened the foam structures capacity of carrying load. Many experimental efforts have been made in recent years to determine the fracture toughness of different types of foams: plastic [4, 5, 6, 7], carbon [8] and metallic [9, 10]. McIntyre and Anderson [11], using single edge notch bend specimens made of rigid closed-cell polyurethane foams, measured the $K_{Ic}$ for different densities. They found that the fracture toughness is independent of crack length and proposed a linear correlation between fracture toughness and density, for foam densities smaller than 200 kg/m$^3$. At higher densities the correlation became non-linear. The same behavior was observed by Danielsson [12] on PVC Divinycell foams and Viana and Carlsson on Diab H foams [5]. Brittle fracture without yielding produced in Mode I was observed in these experiments. It is to be noted that a correlation between the static fracture toughness and relative density $\rho/\rho_s$ was proposed in [1]. Kabir et al. [7] used the procedure described by ASTM D5045 [13] for determining the fracture toughness of polyvinyl chloride (PVC) and polyurethane (PUR) foams. They investigated the effects of density, specimen size, loading rate and of cell orientation. Density has a significant effect on fracture toughness, which increases more than 7 times when the foam density increases 3.5 times. They also presented the results of the established fracture toughness for H130 foams measured with crack orientation in two directions: rise and flow. The fracture toughness is higher with 27% when the crack is orientated parallel to the rise direction. Burman [6] presented fracture toughness results for two commercial foams, Rohacell WF51 (density 52 kg/m$^3$) and Divinycell H100 (density 100 kg/m$^3$). The mode I fracture toughness $K_{Ic}$ was obtained on Single Edge Notch Beam (SENB) specimens and has values 0.08 MPa m$^{0.5}$ for WF51, respectively 0.21 MPa·m$^{0.5}$ for

H100. He also determined the Mode II fracture toughness using End-Notch Flexure (ENF) specimens, with values of 0.13 MPa·m$^{0.5}$ for WF51, respectively 0.21 MPa·m$^{0.5}$ for H100.

The experiments in mixed-mode are done on standard specimens, and one of the most common is the four-point bend specimen. This can create the pure mode I or II and the mixed modes I and II. The four-point bend specimen is loaded in two forms: symmetric and asymmetric. The symmetric bend specimen creates the pure mode I and the mixed mode, but the asymmetric specimen creates mode II in addition to the mixed modes I and II. In [14] a new fundamental reference solution is given for an infinitely long cracked specimen loaded by a constant shear force and the corresponding bending moment. Small corrections need to be applied for a finite four-point loading geometry. The geometry and loading conditions for another improved test configuration called the asymmetric semi-circular bend (ASCB) specimen is presented in [15]. In this case a semi-circular specimen that contains an edge crack emanating normal to the flat edge of the specimen is loaded asymmetrically by a three-point bend fixture. In order to use accurately the analytical solutions for these two testing configurations the loading points have to be sufficiently far from the crack. Various tests under mixed-mode bending for ASCB specimens were presented before in [16].

**Testing geometry**

An asymmetric four-point bend specimen (A4PB) is used in these tests having the geometry presented in Fig. 1. All tested specimens had $B$ = 12.5 mm, $W$ = 25 mm, and $b_1 + b_2$ = 100 mm. An initial geometrical configuration considered $b_1$ = 40 mm, $b_2$ = 60 mm, and $a/W$ = 0.5. For $c$ = 0, Mode I should vanish according to the relations written bellow, from which one can calculate the stress intensity factors for a reference problem with an infinite specimen, [17], subjected to a force $Q$ and a varying bending moment $M$

$$K_I^R = \frac{6cQ}{W^2}\sqrt{\pi a}F_I(a/W) , \quad (1)$$

$$K_{II}^R = \frac{Q}{W^{1/2}}\frac{(a/W)^{3/2}}{(1-a/W)^{1/2}}F_{II}(a/W) . \quad (2)$$

The shear for $Q$ which acts between the inner loading points is given by $Q = P\,(b_2 - b_1)/(b_2 + b_1)$ and $M = cQ$ are force and moment defined per unit thickness. The expressions to calculate $F_I(a/W)$ and $F_{II}(a/W)$ can be found in [17].

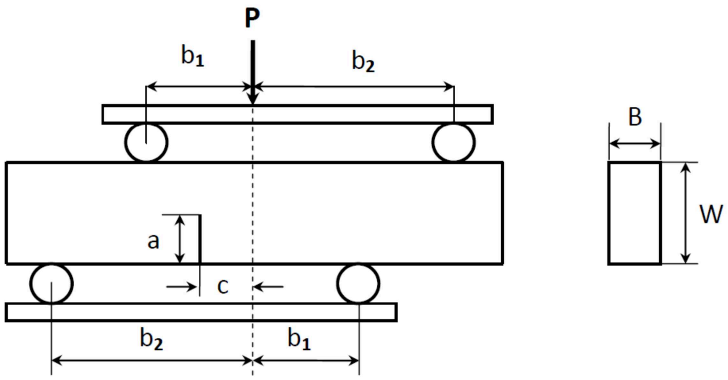


Fig. 1. Geometry of the specimen and loading configuration.

The reference solution of Eqs. 1 and 2 is accurate (finite element results show this in [17]) as long as the distance of the nearest loading point is greater than 1.4$W$. That is $(b_1 - c) > 1.4W$. For our $b_1$ value (initially considered as 40 mm) it results $c$ < 5 mm, as to fulfill this condition. For loading points nearer to the crack, He and Hutchinson [14] established that a correction of the above relations is needed as these are valid only for a reference specimen. Such calculations were done for some geometries and further discussions were presented in [17], as these authors introduced two more correction factors (one for each mode), besides the ones established in [14].

**Results obtained in mode I loading**

In Mode I, three-point bending (3PB) tests were done for $b_1 + b_2 = 100$ mm and the force $P$ applied in the middle. Tests were performed on closed-cell Necuron polyurethane foams of densities 100, 160 and 301 kg/m$^3$. Tests were performed on a Zwick Z010 (10 kN) machine. Speeds of testing were considered as 1, 10, and 100 mm/min. The obtained Mode I average critical toughness is given in Table 1. Average values are obtained for each speed from 4 to 7 tests.

Tabel 1. Mode I fracture toughness for three densities of polyurethane foam.

| **Foam density** [kg/m$^3$] | **Speed of testing** [mm/min] | $K_{Ic}$ $[\mathrm{MPa}\sqrt{\mathrm{m}}]$ |
|---|---|---|
| 100 | 1 | 0.0722 |
| | 10 | 0.0741 |
| | 100 | 0.0735 |
| 160 | 1 | 0.0797 |
| | 10 | 0.0881 |
| | 100 | 0.0861 |
| 301 | 1 | 0.341 |
| | 10 | 0.343 |
| | 100 | not tested |

**Results obtained in mixed mode loading**

As shown in Fig. 1, loading was applied top-down in a four-point bending (A4PB) arrangement, through loading cylinders having a diameter of 10 mm. In Fig. 2 is presented the failed specimen for $c = 0$ - according to Eqs. 1 and 2 only Mode II should be obtained. Due to the loading conditions the foam was crushed severely closer to the crack location and a secondary propagating crack developed (Fig. 2).

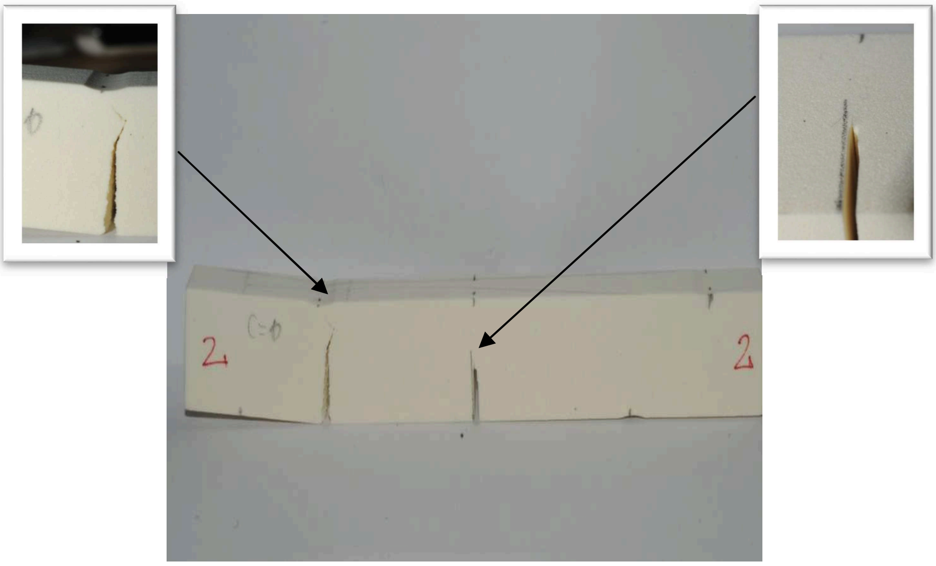

Fig. 2. Failure of an A4PB specimen with $c = 0$.

This crack was the one which finally led to the undesired failure of the specimen.

Almost the same behaviour of the tested specimen resulted when $c = 5$ mm, as presented in Fig. 3. This time the main crack propagated as being oriented towards the nearest loading point, and bifurcated close to the outer surface. One branch turned suddenly to the surface, while the other continued its path to the loading cylinder.

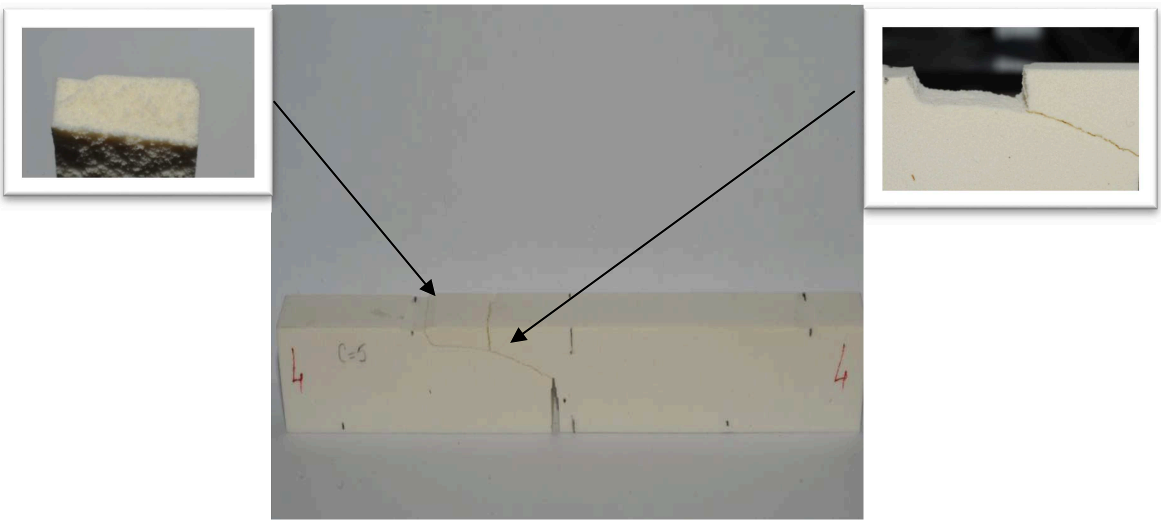


Fig. 3. Failure of an A4PB specimen with $c = 5$ mm.

Although formally the condition $c < 5$ mm discussed previously was preserved at the limit, the local crushing of the polyurethane foam gave the undesired failure of the specimens. In both presented tests the ratio $a/W$ was 0.5, the initial crack being quite close to the top surface of the specimen, where loading is applied. Three sets of geometrical arrangements, named configurations, were considered for the beginning, as presented in Table 2. The values of $c$ result as imposing the previously mentioned condition which should not affect the crack tip due to the close boundaries.

Table 2. Geometry of the specimens and resulting $c$ value.

| **Dimensions** | **Configuration 1** | **Configuration 2** | **Configuration 3** |
|---|---|---|---|
| $b_1$ | 40 mm | 42.5 mm | 45 mm |
| $b_2$ | 60 mm | 57.5 mm | 55 mm |
| $a$ | 12.5 mm | 12.5 mm | 12.5 mm |
| $W$ | 25 mm | 25 mm | 25 mm |
| $B$ | 12.5 mm | 12.5 mm | 12.5 mm |
| It results | $c < 5$ mm | $c < 7.5$ mm | $c < 10$ mm |

Only for the density 160 kg/m$^3$ we present the obtained results in Table 3, which are in fact typical (as difficulty of performing the tests) also for the other two densities. With red colour are marked the tests which failed due to the resons presented previously. For each of the three $b_1$ values the $c$ value is changed, modyfing therefore the mode mixity. In Table 3 dimensions are in [mm], the stress intensity factors are initially calculated as average values between several tests in [MPa·mm$^{0.5}$] and then in [MPa·m$^{0.5}$]. The last two columns give the stress intensity factors in Mode I and Mode II normalized to the critical stress intensity factor established in Mode I.

Table 3. Mixed-mode tests done for the three configurations for density 160 kg/m$^3$.

| $c$ | $b_1$ | $a$ | $W$ | $K_I$ | $K_{II}$ | $F_I$(a/W) | $F_{II}$(a/W) | $K_I$ aver | $K_{II}$ aver | $K_I$ MPam$^{0.5}$ | $K_{II}$ MPam$^{0.5}$ | $K_I/K_{Ic}$ | $K_{II}/K_{Ic}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 40 | 12.43 | 24.67 | 0.191 | 0.716 | 1.506 | 3.411 | 0.191 | 0.716 | 0.00603 | 0.02264 | **0.075** | **0.283** |
| 1 | 40 | x | x | x | x | x | x | | | x | x | | |
| 1 | 40 | x | x | x | x | x | x | | | x | x | | |
| 1 | 40 | x | x | x | x | x | x | | | x | x | | |
| 2.5 | 40 | 12.43 | 24.87 | 0.515 | 0.782 | 1.494 | 3.434 | 0.540 | 0.815 | 0.01627 | 0.02473 | **0.213** | **0.322** |
| 2.5 | 40 | 12.25 | 24.86 | 0.539 | 0.823 | 1.473 | 3.474 | | | 0.01705 | 0.02602 | | |
| 2.5 | 40 | 12.31 | 24.46 | 0.563 | 0.839 | 1.505 | 3.414 | | | 0.01782 | 0.02655 | | |
| 2.5 | 40 | x | x | x | x | x | x | | | x | x | | |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4.5 | 40 | 12.36 | 24.5 | 0.997 | 0.826 | 1.508 | 3.407 | 0.949 | 0.788 | 0.03154 | 0.02614 | **0.375** | **0.312** |
| 4.5 | 40 | 12.19 | 24.55 | 0.963 | 0.804 | 1.484 | 3.452 | | | 0.03046 | 0.025437 | | |
| 4.5 | 40 | 12.3 | 24.42 | 0.888 | 0.734 | 1.506 | 3.411 | | | 0.02808 | 0.02321 | | |
| 4.5 | 40 | x | x | x | x | x | x | | | x | x | | |
| 1 | 42.5 | 12.52 | 24.52 | 0.208 | 0.774 | 1.528 | 3.372 | 0.186 | 0.702 | 0.006593 | 0.0244874 | **0.074** | **0.277** |
| 1 | 42.5 | 12.31 | 24.87 | 0.159 | 0.606 | 1.479 | 3.461 | | | 0.00502 | 0.01916 | | |
| 1 | 42.5 | 12.73 | 24.89 | 0.192 | 0.725 | 1.531 | 3.367 | | | 0.00608 | 0.02291 | | |
| 2.5 | 42.5 | x | x | x | x | x | x | 0.519 | 0.788 | x | x | **0.205** | **0.312** |
| 2.5 | 42.5 | 12.59 | 24.90 | 0.516 | 0.782 | 1.512 | 3.400 | | | 0.01633 | 0.02474 | | |
| 2.5 | 42.5 | 12.52 | 24.97 | 0.521 | 0.795 | 1.499 | 3.424 | | | 0.01649 | 0.02513 | | |
| 4.5 | 42.5 | 12.35 | 24.63 | 0.832 | 0.695 | 1.499 | 3.424 | 0.766 | 0.642 | 0.02630 | 0.02196 | **0.303** | **0.254** |
| 4.5 | 42.5 | 12.25 | 24.58 | 0.830 | 0.694 | 1.489 | 3.442 | | | 0.02627 | 0.02194 | | |
| 4.5 | 42.5 | 12.06 | 24.71 | 0.637 | 0.538 | 1.459 | 3.502 | | | 0.02015 | 0.01703 | | |
| 7 | 42.5 | 12.55 | 24.88 | 1.384 | 0.748 | 1.508 | 3.407 | 1.432 | 0.771 | 0.04376 | 0.02367 | **0.566** | **0.304** |
| 7 | 42.5 | 12.37 | 24.48 | 1.471 | 0.783 | 1.511 | 3.402 | | | 0.04653 | 0.02475 | | |
| 7 | 42.5 | 11.89 | 24.62 | 1.441 | 0.782 | 1.445 | 3.532 | | | 0.04557 | 0.02473 | | |
| 1 | 45 | x | x | x | x | x | x | 0.125 | 0.469 | x | x | **0.049** | **0.185** |
| 1 | 45 | 12.18 | 24.63 | 0.102 | 0.386 | 1.478 | 3.464 | | | 0.00323 | 0.01222 | | |
| 1 | 45 | 12.38 | 24.5 | 0.148 | 0.552 | 1.511 | 3.402 | | | 0.00468 | 0.01746 | | |
| 2.5 | 45 | x | x | x | x | x | x | 0.159 | 0.243 | x | x | **0.063** | **0.096** |
| 2.5 | 45 | x | x | x | x | x | x | | | x | x | | |
| 2.5 | 45 | 12.37 | 24.92 | 0.159 | 0.244 | 1.484 | 3.453 | | | 0.00505 | 0.00771 | | |
| 4.5 | 45 | 12.06 | 24.54 | 0.674 | 0.564 | 1.469 | 3.482 | 0.650 | 0.548 | 0.02131 | 0.01785 | **0.257** | **0.217** |
| 4.5 | 45 | 12.27 | 24.9 | 0.559 | 0.475 | 1.473 | 3.474 | | | 0.01769 | 0.01502 | | |
| 4.5 | 45 | 12.47 | 24.89 | 0.717 | 0.605 | 1.498 | 3.427 | | | 0.02269 | 0.01915 | | |
| 7 | 45 | 12.11 | 24.82 | 0.974 | 0.531 | 1.459 | 3.503 | 0.955 | 0.522 | 0.03076 | 0.01678 | **0.377** | **0.206** |
| 7 | 45 | 11.82 | 24.88 | 1.076 | 0.592 | 1.424 | 3.578 | | | 0.03404 | 0.01874 | | |
| 7 | 45 | 12.5 | 24.89 | 0.816 | 0.442 | 1.501 | 3.420 | | | 0.02580 | 0.01398 | | |
| 9.5 | 45 | 12.17 | 24.49 | 1.358 | 0.535 | 1.485 | 3.450 | 1.422 | 0.563 | 0.04293 | 0.01693 | **0.562** | **0.222** |
| 9.5 | 45 | 12.29 | 24.71 | 1.355 | 0.539 | 1.486 | 3.447 | | | 0.04285 | 0.01705 | | |
| 9.5 | 45 | 12.61 | 24.75 | 1.553 | 0.613 | 1.524 | 3.378 | | | 0.04911 | 0.01939 | | |

## Conclusions

The normalized values of the SIFs are the ones which can be plotted as to represent the locus of an failure envelope established experimentally (for a detailed discussion see [16]) which can be compared to other mixed-mode criteria already known in the literature.

The tests were also monitored using the digital image correlation (DIC) method which revealed the local strains at the tip of the crack before propagation. The angle at which the crack initiates to propagate can be also established experimentally.

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