Cele mai relevante 10 lucrări CĂLIN-ADRIAN POPA

- 1. Petre Birtea, Dan Comănescu, Călin-Adrian Popa, *Averaging on Manifolds by Embedding Algorithm*. Journal of Mathematical Imaging and Vision, Vol. 49, Nr. 2, pag. 454-466, 2014. (ISI Impact Factor 1.353, Q2)
- Călin-Adrian Popa, Learning Algorithms for Quaternion-Valued Neural Networks. Neural Processing Letters, Vol. 47, Nr. 3, pag. 949–973, 2018. (ISI Impact Factor 2.891, Q2)
- 3. Călin-Adrian Popa, Eva Kaslik, *Multistability and multiperiodicity in impulsive hybrid quaternion-valued neural networks with mixed delays*. **Neural Networks**, Vol. 99, pag. 1-18, 2018. (ISI Impact Factor 5.535, Q1)
- Călin-Adrian Popa, Global exponential stability of octonion-valued neural networks with leakage delay and mixed delays. Neural Networks, Vol. 105, pag. 277-293, 2018. (ISI Impact Factor 5.535, Q1)
- Călin-Adrian Popa, Global exponential stability of neutral-type octonion-valued neural networks with time-varying delays. Neurocomputing, Vol. 309, pag. 117-133, 2018. (ISI Impact Factor 4.438, Q1)
- Călin-Adrian Popa, Global μ-stability of neutral-type impulsive complex-valued BAM neural networks with leakage delay and unbounded time-varying delays. Neurocomputing, Vol. 376, pag. 73-94, 2020. (ISI Impact Factor 4.438, Q1)
- 7. Călin-Adrian Popa, *Dissipativity of impulsive matrix-valued neural networks with leakage delay and mixed delays*. **Neurocomputing**, Vol. 405, pag. 85-95, 2020. (ScienceDirect, ISI Impact factor 4.438, Q1)
- Călin-Adrian Popa, Eva Kaslik, Finite-Time Mittag–Leffler Synchronization of Neutral-Type Fractional-Order Neural Networks with Leakage Delay and Time-Varying Delays. Mathematics, Vol. 8, No. 7, 2020. (MDPI, ISI Impact factor 1.747, Q1)
- 9. Călin-Adrian Popa, Quasi-Newton Learning Methods for Complex-Valued Neural Networks, 29th International Joint Conference on Neural Networks (IJCNN), 2015, Killarney, Ireland. (IEEE, ISI)
- 10. Călin-Adrian Popa, *Complex-Valued Convolutional Neural Networks for Real-Valued Image Classification*, 30th International Joint Conference on Neural Networks (IJCNN), pag. 816-822, 2017, Anchorage, Alaska, USA. (IEEE, ISI)

Averaging on Manifolds by Embedding Algorithm

Petre Birtea · Dan Comănescu · Călin-Adrian Popa

Published online: 3 December 2013 © Springer Science+Business Media New York 2013

Abstract We will propose a new algorithm for finding critical points of cost functions defined on a differential manifold. We will lift the initial cost function to a manifold that can be embedded in a Riemannian manifold (Euclidean space) and will construct a vector field defined on the ambient space whose restriction to the embedded manifold is the gradient vector field of the lifted cost function. The advantage of this method is that it allows us to do computations in Cartesian coordinates instead of using local coordinates and covariant derivatives on the initial manifold. We will exemplify the algorithm in the case of SO(3) averaging problems and will rediscover a few well known results that appear in literature.

Keywords Averaging · Optimization · Distance functions · Rotations · Quaternions · Metriplectic dissipation · Gradient vector field

1 Introduction

A problem which one frequently encounters in applications is to find an average for a finite set of sample points $\{y_1, y_2, \ldots, y_r\}$ belonging to a manifold *N*. A method to

D. Comănescu e-mail: comanescu@math.uvt.ro

C.-A. Popa

solve this problem is to write it as an optimization problem, more precisely to construct a cost function $G_N : N \to \mathbb{R}$ associated with this set of sample points and then the average is defined by

 $\arg\min_{y\in N}G_N(y).$

Of special interest are the cost functions of least square type $G_N(y) := \sum_{i=1}^r d^2(y, y_i)$, where *d* is a distance function on *N*. The average of the set of sample points $\{y_1, y_2, \dots, y_r\}$ on the manifold *N* is the set defined by

$$\arg\min_{y\in N}\sum_{i=1}^r d^2(y, y_i).$$

When the function G_N is differentiable, this is equivalent with solving the equation $dG_N(y) = 0$ and with testing for which solutions the minimum value is attained. In order to write this equation we need the knowledge of a local system of coordinates on the manifold N, a requirement that might be difficult to fulfill in many practical cases. Another problem that we may encounter with this approach is that the set of sample points $\{y_1, \ldots, y_r\}$ might not be entirely included in the domain of a single local system of coordinates. One way to overcome these difficulties is to lift the problem on a simpler manifold S.

Let $\mathbf{P}: S \to N$ be a surjective submersion. The lifted cost function is $G_S: S \to \mathbb{R}$ defined by $G_S = G_N \circ \mathbf{P}$. The set equality $\mathbf{P}(\{x \in S \mid dG_S(x) = 0\}) = \{y \in N \mid dG_N(y) = 0\}$ shows that it is sufficient to solve the equation $dG_S(x) = 0$ on the simpler manifold *S* and project these solutions on the manifold *N*.

A way to solve this new problem is to endow S with a Riemannian metric τ and compute the equilibrium points of the gradient vector field $\nabla_{\tau} G_S$ and verify which of them are

P. Birtea (🖂) · D. Comănescu

Department of Mathematics, West University of Timişoara, Bd. V. Pârvan, No. 4, 300223 Timişoara, Romania e-mail: birtea@math.uvt.ro

Department of Computer Science, "Politehnica" University of Timişoara, Bd. V. Pârvan, No. 2, 300223 Timişoara, Romania e-mail: calin.popa@cs.upt.ro



Learning Algorithms for Quaternion-Valued Neural Networks

Călin-Adrian Popa¹

Published online: 25 September 2017 © Springer Science+Business Media, LLC 2017

Abstract This paper presents the deduction of the enhanced gradient descent, conjugate gradient, scaled conjugate gradient, quasi-Newton, and Levenberg–Marquardt methods for training quaternion-valued feedforward neural networks, using the framework of the HR calculus. The performances of these algorithms in the real- and complex-valued cases led to the idea of extending them to the quaternion domain, also. Experiments done using the proposed training methods on time series prediction applications showed a significant performance improvement over the quaternion gradient descent algorithm.

Keywords Quaternion-valued neural networks \cdot Quickprop \cdot Resilient backpropagation \cdot Delta-bar-delta \cdot SuperSAB \cdot Conjugate gradient algorithms \cdot Scaled conjugate gradient algorithm \cdot Quasi-Newton algorithms \cdot Levenberg–Marquardt algorithm \cdot Time series prediction

1 Introduction

The domain of quaternion-valued neural networks has received an increasing interest over the last few years. Some popular applications of these networks include chaotic time-series prediction [4], color image compression [23], color night vision [27], polarized signal classification [9], and 3D wind forecasting [25,52,54].

Some signals in the 3D and 4D domains can be more naturally expressed in quaternionvalued form. Thus, these networks appear as a natural choice for solving problems such as time series prediction. Several methods have been proposed to increase the efficiency of learning in quaternion-valued neural networks. These methods include different network architectures and different learning algorithms, some of which are specially designed for this type of networks, while others are extended from the real-valued case.

Călin-Adrian Popa calin.popa@cs.upt.ro

¹ Department of Computer and Software Engineering, Polytechnic University Timişoara, Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania

ELSEVIER

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Multistability and multiperiodicity in impulsive hybrid quaternion-valued neural networks with mixed delays

Călin-Adrian Popa^a, Eva Kaslik^{b,c,*}

^a Department of Computer and Software Engineering, Polytechnic University Timişoara, Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania

^b Institute e-Austria Timișoara, Blvd. V. Pârvan nr. 4, room 045B, 300223, Timișoara, Romania

^c Department of Mathematics and Computer Science, West University of Timisoara, Blvd. V. Parvan nr. 4, 300223, Timisoara, Romania

HIGHLIGHTS

- Impulsive QVNNs with time-dependent and distributed delays are considered.
- Sufficient conditions are obtained for the existence of 16ⁿ attractors.
- Improved storage capacity over real- or complex-valued neural networks is achieved.
- Multistability and multiperiodicity properties of attractors are investigated.
- Numerical examples are given to illustrate the theoretical results.

ARTICLE INFO

Article history: Received 10 August 2017 Received in revised form 25 October 2017 Accepted 12 December 2017 Available online 18 December 2017

Keywords: Quaternion-valued neural networks Distributed delays Impulses Multistability Periodic solutions Multiperiodicity

ABSTRACT

The existence of multiple exponentially stable equilibrium states and periodic solutions is investigated for Hopfield-type quaternion-valued neural networks (QVNNs) with impulsive effects and both timedependent and distributed delays. Employing Brouwer's and Leray–Schauder's fixed point theorems, suitable Lyapunov functionals and impulsive control theory, sufficient conditions are given for the existence of 16ⁿ attractors, showing a substantial improvement in storage capacity, compared to real-valued or complex-valued neural networks. The obtained criteria are formulated in terms of many adjustable parameters and are easily verifiable, providing flexibility for the analysis and design of impulsive delayed QVNNs. Numerical examples are also given with the aim of illustrating the theoretical results.

© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

Quaternion-valued neural networks (QVNNs) are natural extensions of the extensively studied real-valued neural networks (RVNNs) and complex-valued neural networks (CVNNs) (Hirose, 2009). While two-dimensional data can be successfully operated by complex-valued neurons, QVNNs are expected to be more adequate to process multidimensional information (e.g., threedimensional coordinates and color), by means of quaternionic neurons.

Quaternions have first been introduced by William Rowan Hamilton in 1843, the algebra of quaternions being the first noncommutative division algebra to be discovered. Quaternions

* Corresponding author at: Department of Mathematics and Computer Science, West University of Timişoara, Blvd. V. Parvan nr. 4, 300223, Timişoara, Romania.

E-mail addresses: calin.popa@cs.upt.ro (C.-A. Popa), eva.kaslik@e-uvt.ro (E. Kaslik).

https://doi.org/10.1016/j.neunet.2017.12.006 0893-6080/© 2017 Elsevier Ltd. All rights reserved. attracted attention from various areas, such as attitude control, computer graphics, and quantum mechanics, due to their efficiency in modeling multidimensional data.

Quaternionic multilayer perceptrons have first been introduced by Arena, Fortuna, Occhipinti, and Xibilia (1994), proving their efficiency in quaternionic function interpolation and chaotic time series prediction (Arena, Baglio, Fortuna, & Xibilia, 1995). Applications of QVNNs include color image compression (Isokawa, Kusakabe, Matsui, & Peper, 2003) and color night vision (Kusamichi, Isokawa, Matsui, Ogawa, & Maeda, 2004) as well, where RGB values are represented by the three imaginary parts of the quaternion. Moreover, QVNNs have also been used in 3D wind field modeling and forecasting (Took, Strbac, Aihara, & Mandic, 2011), where the three perpendicular wind speeds and temperature have been modeled as a full quaternion-valued quantity. Compared to RVNNs, Buchholz and Le Bihan (2006) showed that QVNNs provide improved performance in polarized signal classification.



ELSEVIER

Contents lists available at ScienceDirect

Neural Networks



journal homepage: www.elsevier.com/locate/neunet

Global exponential stability of octonion-valued neural networks with leakage delay and mixed delays



Călin-Adrian Popa

Department of Computer and Software Engineering, Polytechnic University Timişoara, Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania

ARTICLE INFO

Article history: Received 19 November 2017 Received in revised form 12 March 2018 Accepted 4 May 2018 Available online 14 June 2018

Keywords: Octonion-valued neural networks Distributed delays Global stability Linear matrix inequalities

ABSTRACT

This paper discusses octonion-valued neural networks (OVNNs) with leakage delay, time-varying delays, and distributed delays, for which the states, weights, and activation functions belong to the normed division algebra of octonions. The octonion algebra is a nonassociative and noncommutative generalization of the complex and quaternion algebras, but does not belong to the category of Clifford algebras, which are associative. In order to avoid the nonassociativity of the octonion algebra and also the noncommutativity of the quaternion algebra, the Cayley–Dickson construction is used to decompose the OVNNs into 4 complex-valued systems. By using appropriate Lyapunov–Krasovskii functionals, with double and triple integral terms, the free weighting matrix method, and simple and double integral Jensen inequalities, delay-dependent criteria are established for the exponential stability of the considered OVNNs. The criteria are given in terms of complex-valued linear matrix inequalities, for two types of Lipschitz conditions which are assumed to be satisfied by the octonion-valued activation functions. Finally, two numerical examples illustrate the feasibility, effectiveness, and correctness of the theoretical results.

© 2018 Elsevier Ltd. All rights reserved.

1. Introduction

Neural networks with values in multidimensional domains have attracted the attention of researchers over the last few years. First introduced by Widrow, McCool, and Ball (1975), complexvalued neural networks (CVNNs) have found numerous applications, which include antenna design, radar imaging, estimation of direction of arrival and beamforming, image processing, communications signal processing, and many others (Hirose, 2012, 2013). Quaternion-valued neural networks (QVNNs) were introduced by Arena, Fortuna, Occhipinti, and Xibilia (1994), and have applications in chaotic time-series prediction (Arena, Fortuna, Muscato, & Xibilia, 1998), color image compression (Isokawa, Kusakabe, Matsui, & Peper, 2003), color night vision (Kusamichi, Isokawa, Matsui, Ogawa, & Maeda, 2004), polarized signal classification (Buchholz & Le Bihan, 2008), and 3D wind forecasting (Jahanchahi, Took, & Mandic, 2010; Took, Mandic, & Aihara, 2010). Cliffordvalued neural networks (ClVNNs), proposed by Pearson and Bisset (1992, 1994), and later discussed by Buchholz and Sommer (2008) and Kuroe, Tanigawa, and Iima (2011), have potential applications in high-dimensional data processing. They represent a generalization of the complex- and quaternion-valued neural networks, because complex and quaternion algebras are special cases of the 2^n -dimensional Clifford algebras, where $n \ge 1$.

https://doi.org/10.1016/j.neunet.2018.05.006 0893-6080/© 2018 Elsevier Ltd. All rights reserved. A different generalization of the complex and quaternion algebras is the octonion algebra. It is an 8-dimensional normed division algebra, which means that a norm and a multiplicative inverse can be defined on it. In fact, it is the only normed division algebra that can be defined over the field of real numbers, besides the complex and quaternion algebras. The octonion algebra is not a special kind of Clifford algebra, because the Clifford algebras are all associative, whereas the octonion algebra is not.

Octonions have applications in physics and geometry (Dray & Manogue, 2015; Okubo, 1995), and have also been successfully applied in the signal processing domain in the recent years (Snopek, 2015). The signal processing applications include salient object detection (Gao & Lam, 2014a, b), hyperspectral fluorescence data fusion (Bauer & Leon, 2016), L1-norm minimization for octonion signals (Wang, Xiang, & Zhang, 2016), and the octonion Fourier transform (Błaszczyk & Snopek, 2017). In physics, octonions were used to reformulate electrodynamics and chromodynamics equations (Chanyal, 2013; Chanyal, Bisht, Li, & Negi, 2012; Chanyal, Bisht, & Negi, 2010, 2011), the Maxwell equations (Demir & Tanişli, 2016), the gravitational field equations (Demir, 2012), and the Dirac equation (Koplinger, 2006).

Taking all the above facts into consideration, octonions may have potential applications in the neural network domain, also. Thus, feedforward octonion-valued neural networks (OVNNs) were first proposed by Popa (2016). They may be applied in the signal processing domain, where certain signals can be better

E-mail address: calin.popa@cs.upt.ro.

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Global exponential stability of neutral-type octonion-valued neural networks with time-varying delays

Călin-Adrian Popa

Department of Computer and Software Engineering, Polytechnic University Timişoara, Blvd. V. Pârvan, No. 2, Timişoara 300223, Romania

ARTICLE INFO

Article history: Received 26 November 2017 Revised 5 March 2018 Accepted 3 May 2018 Available online 9 May 2018

Communicated by Qiankun Song

Keywords: Octonion-valued neural networks Neutral-type neural networks Global stability Linear matrix inequalities

ABSTRACT

Octonion-valued neural networks (OVNNs) are a type of neural networks for which the states and weights are octonions. The octonion algebra is the only normed division algebra that can be defined over the reals, besides the complex and quaternion algebras. Being nonassociative, it clearly does not belong to the Clifford algebras category, which are all associative. In this paper, sufficient conditions for the global exponential stability of neutral-type OVNNs with time-varying delays are formulated, by considering two types of Lipschitz conditions that must be satisfied by the octonion-valued activation functions. To avoid the nonassociativity of the octonions and the noncommutativity of the quaternions, the OVNNs model is decomposed into 4 complex-valued systems, using the Cayley–Dickson construction. By using Lyapunov–Krasovskii functionals with double, triple, and quadruple integral terms, the free weighting matrix method, and simple, double, and triple Jensen inequalities, the stability criteria are formulated in terms of complex-valued linear matrix inequalities. Two numerical examples are provided in order to demonstrate the effectiveness and feasibility of the theoretical results.

© 2018 Elsevier B.V. All rights reserved.

1. Introduction

Multidimensional neural networks have attracted interest especially over the last few years. The most developed are the complexvalued neural networks (CVNNs), which were first proposed by Widrow et al. [1], and found applications in radar imaging, image processing, communications signal processing, antenna design, estimation of direction of arrival and beam forming, and many others [2,3]. Another type of multidimensional networks are the quaternion-valued neural networks (QVNNs), which were proposed by Arena et al. [4], but became more popular in the very recent years. Their applications include chaotic time-series prediction [5], color image compression [6], color night vision [7], polarized signal classification [8], three-dimensional wind forecasting [9,10], and others that are continuously appearing. Both types of networks have been generalized to Clifford-valued neural networks (CIVNNs), which were introduced by Pearson and Bisset [11,12], and studied by Buchholz et al. [13,14]. They are defined on the 2^n -dimensional Clifford algebras, $n \ge 1$, of which the complex and quaternion algebras are special cases. It is expected that, in the future, CIVNNs will have applications in high-dimensional data processing.

Another type of generalization of the complex and quaternion algebras is the octonion algebra, which is 8-dimensional. It has

https://doi.org/10.1016/j.neucom.2018.05.004 0925-2312/© 2018 Elsevier B.V. All rights reserved. the important property of being a normed division algebra, which means that a norm and a multiplicative inverse can be defined on it. In fact, it can be proved that the complex, quaternion, and octonion algebras are the only normed division algebras that can be defined over the field of real numbers. The octonion algebra is noncommutative, but also nonassociative, which shows that it is not a special case of the Clifford algebras, because these algebras are all associative.

Octonions have found applications, among others, in geometry [15], physics [16], and signal processing [17]. In physics, they were used as a means to reformulate the Dirac equation [18], electrody-namics and chromodynamics equations [19–22], the gravitational field equations [23], and the Maxwell equations [24]. Recently, octonions have gained popularity in the signal processing domain, where they were applied to image saliency detection [25,26], L1-norm minimization [27], hyperspectral fluorescence data fusion [28], and the octonion Fourier transform [29].

Taking all the above facts into consideration, it is natural to also apply octonions to the neural network domain. As such, feedforward octonion-valued neural networks (OVNNs) were first introduced by Popa [30]. They have potential applications in signal processing and high-dimensional data processing.

The dynamic properties of neural networks are important for their practical applications. In problems such as neural control, optimization, and signal processing, neural networks must be designed so that they exhibit a single steady state. Thus, it is





E-mail address: calin.popa@cs.upt.ro

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Global μ -stability of neutral-type impulsive complex-valued BAM neural networks with leakage delay and unbounded time-varying delays

Călin-Adrian Popa

Department of Computer and Software Engineering, Polytechnic University Timișoara, Blvd. V. Pârvan, No. 2, Timișoara 300223, Romania

ARTICLE INFO

Article history: Received 6 December 2018 Revised 30 July 2019 Accepted 6 September 2019 Available online 13 September 2019

Communicated by Prof. Qiankun Song

Keywords: Complex-valued BAM neural networks Neutral-type neural networks Global stability Impulses Linear matrix inequalities

ABSTRACT

This paper investigates the existence and uniqueness of the equilibrium point and its global μ -stability for neutral-type impulsive complex-valued bidirectional associative memory neural networks with leakage delay and unbounded time-varying delays, by considering two types of Lipschitz conditions to be satisfied by the activation functions. The homeomorphism lemma is used to obtain a sufficient condition for the existence and uniqueness of the equilibrium point. Sufficient delay-dependent conditions both in terms of complex-valued and real-valued linear matrix inequalities which ensure the global μ -stability of the equilibrium point are obtained by constructing appropriate Lyapunov–Krasovskii functionals with simple, double, and triple integral terms, and using the free weighting matrix method, simple and double complex-valued Jensen inequalities, the complex-valued reciprocally convex combination inequality, and the complex-valued Wirtiger-based integral inequality. Lastly, two numerical examples are provided to illustrate the effectiveness of the obtained theoretical results.

© 2019 Elsevier B.V. All rights reserved.

1. Introduction

Over the last few years, there has been an increasing interest in the domain of recurrent neural networks, especially the following types of models: Hopfield [1,2], Cohen–Grossberg [3], cellular [4,5], and bidirectional associative memory neural networks [6], mainly because of their applications in many areas such as classification, optimization, signal and image processing, solving nonlinear algebraic equations, pattern recognition, system identification, associative memories, cryptography, and so on. These applications are highly dependent on the dynamical properties of the networks. Thus, the analysis of the dynamical behavior is an important part in the design of the recurrent neural networks used in applications.

The study of the dynamics of recurrent neural networks has become a field of study in its own right, attracting many researchers. The review paper [7] lists more than 300 references only for the stability analysis of continuous-time recurrent neural networks, not taking into account the discrete-time ones, or other dynamical properties that can be studied such as bifurcation, attractivity, dissipativity, passivity, synchronization, and so on.

Since they were first introduced by Kosko in [6], bidirectional associative memories, an extension of the unidirectional auto-

associative Hopfield neural networks, were intensely studied, and have many applications in pattern recognition, signal and image processing, and automatic control.

On the other hand, complex-valued neural networks have been proposed by Aizenberg et al. [8], but have caught the attention of researchers in the past years, especially due to their applications in physical systems dealing with electromagnetic, ultrasonic, quantum, and light waves, and also in filtering, imaging, optoelectronics, speech synthesis, computer vision, and so on (see, for example, [9,10]). In these applications, the stability of the complex-valued neural networks plays a very important role. Moreover, they have more complicated properties than the real-valued neural networks because of their complex-valued states, connection weights, and activation functions. The activation functions cannot be a simple generalization of the real-valued ones, because, by Liouville's theorem, it can be deduced that a bounded entire function is a constant, which makes the choice of such functions more difficult. As a consequence, the study of the dynamic behavior of the complexvalued recurrent neural networks has received increasing interest, especially in the last few years.

Because time delays inherently occur in real life implementations of neural networks, and they can lead to unwanted behavior such as oscillations and chaos, complex-valued neural networks with different types of time delays have been proposed and studied. From the various types of delays we mention: constant







E-mail address: calin.popa@cs.upt.ro

https://doi.org/10.1016/j.neucom.2019.09.008 0925-2312/© 2019 Elsevier B.V. All rights reserved.

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Dissipativity of impulsive matrix-valued neural networks with leakage delay and mixed delays



Călin-Adrian Popa

Department of Computer and Software Engineering Polytechnic University Timişoara Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania

ARTICLE INFO

Article history: Received 17 December 2019 Revised 24 February 2020 Accepted 20 March 2020 Available online 27 March 2020

Communicated by Prof. Qiankun Song

Keywords: Matrix-valued neural networks Leakage delay Mixed delays Dissipativity Linear matrix inequalities

ABSTRACT

A generalization of real-, complex-, and quaternion-valued neural networks is represented by matrixvalued neural networks (MVNNs), for which the states and weights are matrices. The dissipativity of impulsive MVNNs with leakage delay and mixed delays is studied in this paper, by giving sufficient criteria expressed in terms of real-valued linear matrix inequalities. After decomposing the MVNNs into real-valued systems, Lyapunov–Krasovskii functionals with double, triple, and quadruple integral terms are formulated. Also, the free weighting matrix method, simple, double, and triple Jensen inequalities, the reciprocally convex combination inequality, and the Wirtiger-based integral inequality are used to establish the sufficient criteria. Two numerical examples illustrate the feasibility and correctness of the proposed theoretical results.

© 2020 Elsevier B.V. All rights reserved.

1. Introduction

Multidimensional neural networks have caught the attention of researchers over the last few years. Complex-valued neural networks (CVNNs), first introduced in [1], have numerous applications, including radar imaging, antenna design, image processing, estimation of direction of arrival and beamforming, communications signal processing, and many others [2,3]. Hyperbolic numbers, which also form a 2-dimensional algebra, represent the basis for hyperbolic-valued neural networks (HVNNs), which are another type of multidimensional networks [4,5]. First introduced in [6], guaternion-valued neural networks (OVNNs) were applied in color image compression [7], polarized signal classification [8], color night vision [9], chaotic time-series prediction [10], and 3D wind forecasting [11,12]. Proposed in [13,14], and later discussed in [4,15], were Clifford-valued neural networks (ClVNNs), which have potential applications in high-dimensional data processing. These networks represent a generalization of CVNNs, HVNNs, and QVNNs, because complex, hyperbolic, and quaternion algebras are special cases of the 2^n -dimensional Clifford algebras, where $n \ge 1$. Lastly, octonion-valued neural networks (OVNNs), which don't fall into the Clifford neural networks category, were proposed in [16].

The dissipativity of CVNNs was studied in the recent years. Dissipativity for memristor-based CVNNs with time-varying delays was discussed in [17] and [18]. Dissipativity and stability analysis

https://doi.org/10.1016/j.neucom.2020.03.042 0925-2312/© 2020 Elsevier B.V. All rights reserved. for fractional-order CVNNs with time delay was undergone in [19], and for BAM CVNNs with time delay in [20]. The matrix measure method was used in [21] to study the dissipativity of discontinuous delayed CVNNs. Stochastic CVNNs with time-varying delays were analyzed from the dissipativity point of view in [22]. On the other hand, the synchronization of memristive CVNNs was discussed in [23], the anti-synchronization of memristive CVNNs in [24], and the adaptive synchronization of memristive CVNNs in [25].

For QVNNs, the global dissipativity for models with delay was studied in [26]. Memristor-based QVNNs with proportional delay were the focus of [27], where the global dissipativity of such networks was discussed. A criterion for the global dissipativity of a class of BAM QVNNs with time delay was given in [28].

CIVNNs began to be studied only very recently. Using the decomposition of the Clifford numbers into their real components, the global stability of CIVNNs with time delays was discussed in [29]. The global exponential stability of CIVNNs without delays was investigated in [30], directly in the Clifford domain. The existence and global exponential stability of anti-periodic solution for CIVNNs was studied in [31], and the existence and global exponential stability of pseudo almost periodic solution for CIVNNs in [32]. No dissipativity results exist for CIVNNs in the literature, to the best of our knowledge.

Lastly, the exponential stability of OVNNs with leakage delay and mixed delays was discussed in [33], and the exponential stability for neutral-type OVNNs with time-varying delays in [34]. As for ClVNNs, the dissipativity of OVNNs has not yet been discussed in the existing literature.



E-mail address: calin.popa@cs.upt.ro



Article

Finite-Time Mittag–Leffler Synchronization of Neutral-Type Fractional-Order Neural Networks with Leakage Delay and Time-Varying Delays

Călin-Adrian Popa ^{1,2,†} and Eva Kaslik ^{3,*,†}

- ¹ Department of Mathematics, West University of Timişoara, Blvd. V. Pârvan, No. 4, 300223 Timişoara, Romania; calin.popa@cs.upt.ro
- ² Department of Computer and Software Engineering, Polytechnic University Timişoara, Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania
- ³ Department of Computer Science, West University of Timişoara, Blvd. V. Pârvan, No. 4, 300223 Timişoara, Romania
- * Correspondence: eva.kaslik@e-uvt.ro
- + These authors contributed equally to this work.

Received: 15 June 2020; Accepted: 9 July 2020; Published: 13 July 2020



Abstract: This paper studies fractional-order neural networks with neutral-type delay, leakage delay, and time-varying delays. A sufficient condition which ensures the finite-time synchronization of these networks based on a state feedback control scheme is deduced using the generalized Gronwall–Bellman inequality. Then, a different state feedback control scheme is employed to realize the finite-time Mittag–Leffler synchronization of these networks by using the fractional-order extension of the Lyapunov direct method for Mittag–Leffler stability. Two numerical examples illustrate the feasibility and the effectiveness of the deduced sufficient criteria.

Keywords: fractional-order neural networks; finite-time synchronization; neutral-type neural networks; leakage delay; Mittag–Leffler function

1. Introduction

Fractional calculus studies the different possibilities of defining real or complex orders for the differentiation and integration operators. Although it has a long history, only recently it has been successfully applied to physics and engineering problems. As such, in the past few years, it became clear for engineers and scientists that some phenomena can be more accurately described by employing the fractional derivative. Fractional differential equations have been proved to better describe many systems in interdisciplinary fields, such as chemistry, biology, physics, mechanics, electromagnetism, heat transfer, acoustics, economy, and finance.

Fractional-order systems have been proven to possess infinite memory. Taking this fact into account, an extremely important improvement would be the introduction of a memory term (represented by a fractional derivative or integral) into a neural network model. Thus, fractional-order artificial neural networks were developed in [1]. Since then, many properties of this type of networks were studied: asymptotic stability and synchronization [2–7], Mittag–Leffler stability and synchronization [8–13], dissipativity [14–16], etc.

The finite-time stability and synchronization properties of fractional-order neural networks were intensely studied over the last few years. Concretely, "finite-time stability analysis of fractional-order complex-valued memristor-based neural networks with time delays" was done in [17]. Finite-time stability criteria for fractional-order delayed neural networks were also established in [18–20]. A more general model, namely fractional-order Cohen–Grossberg BAM neural networks with time delays was researched in [21], from the finite-time stability point of view. Finite-time stability analysis



Quasi-Newton Learning Methods for Complex-Valued Neural Networks

Călin-Adrian Popa Department of Computer and Software Engineering Polytechnic University Timişoara Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania Email: calin.popa@cs.upt.ro

Abstract—This paper presents the full deduction of the quasi-Newton learning methods for complex-valued feedforward neural networks. Since these algorithms yielded better training results for the real-valued case, an extension to the complexvalued case is a natural option to enhance the performance of the complex backpropagation algorithm. The training methods are exemplified on various well-known synthetic and realworld applications. Experimental results show a significant improvement over the complex gradient descent algorithm.

I. INTRODUCTION

Over the last few years, the domain of complex-valued neural networks has received an increasing interest. Some popular applications of these networks include antenna design, estimation of direction of arrival and beamforming, radar imaging, communications signal processing, image processing, and many others (for an extensive presentation of the applications of complex-valued neural networks, see [1]).

In the signal processing domain, where some signals are naturally expressed in complex-valued form, these networks appear as a natural choice for solving problems such as channel equalization or time series prediction. Several methods have been proposed to increase the efficiency of learning in complex-valued neural networks. These methods include different network architectures and different learning algorithms, some of which are specially designed for this type of networks, while others are extended from the real-valued case.

One such method, which has proven to be very efficient in many applications, is the quasi-Newton learning method. First proposed, among others, by [2], [3], quasi-Newton method has become today one of the most known and used methods to train feedforward neural networks. Having this idea in mind, it seems natural to extend this learning algorithm to complex-valued neural networks, also.

In this paper, we present the full deduction of the most known and used variants of the quasi-Newton method. We also give a general method to compute the gradient of the error function that works either for fully complex, or for split complex activation functions, and for a general multilayer feedforward complex-valued neural network. We test the proposed quasi-Newton methods on both synthetic and realworld applications. The synthetic applications include two fully complex function approximation problems and two split complex function approximation problems. The real-world applications include linear and nonlinear channel equalization problems as well as linear and nonlinear signal prediction problems.

The remainder of this paper is organized as follows: Section II gives a thorough explanation of the quasi-Newton methods for calculating the approximation of the Hessian and inverse Hessian for the optimization of an error function defined on the complex plane. The experimental results of eight applications of the proposed algorithms are shown and discussed in Section III, along with a detailed description of the nature of each problem. Section IV is dedicated to presenting the conclusions of this study.

II. QUASI-NEWTON LEARNING

From the second order methods used to minimize the error function, the Newton method is in theory one of the most effective, see [4]. But because it needs the explicit calculation of the Hessian matrix of the error function, more precisely its inverse, and because this is a computationally expensive task, quasi-Newton methods have been developed. These methods replace the explicit calculation of the Hessian with an approximation of it, which is positive definite by construction, to avoid the convergence problems in the Newton method when the Hessian is not positive definite.

We will use the framework of Wirtinger calculus, or \mathbb{CR} calculus (for a complete survey, see [5]). Let $E : \mathbb{C}^N \to \mathbb{R}$ be the error function of a complex-valued feedforward neural network. In this type of calculus, we need to define the function $E' : \mathbb{C}^{2N} \to \mathbb{R}$ by $E'(\mathbf{w}') = E(\mathbf{w})$, where $\mathbf{w}' = \begin{pmatrix} \mathbf{w} \\ \overline{\mathbf{w}} \end{pmatrix} \in \mathbb{C}^{2N}$, and $\overline{\mathbf{w}}$ is the complex conjugate of the vector $\mathbf{w} \in \mathbb{C}^N$ of all the weights and biases of the network. In what follows, by abuse of notation, we will use E instead of E', the discrimination between the two functions being given by their arguments.

The Newton method gives the following update rule for the vector $\mathbf{w}' \in \mathbb{C}^{2N}$:

$$\mathbf{w}_{k+1}' = \mathbf{w}_k' - (\nabla^2 E(\mathbf{w}_k'))^{-1} \nabla E(\mathbf{w}_k'),$$

where by ∇E we denoted the gradient of the error function E and by $\nabla^2 E$ its Hessian matrix.

In the quasi-Newton methods, we replace the inverse Hessian matrix $(\nabla^2 E(\mathbf{w}'_k))^{-1}$ by a matrix denoted by \mathbf{H}_{k+1} ,

Complex-Valued Convolutional Neural Networks for Real-Valued Image Classification

Călin-Adrian Popa Department of Computer and Software Engineering Polytechnic University Timişoara Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania E-mail: calin.popa@cs.upt.ro

Abstract—In this paper, complex-valued convolutional neural networks are presented, by giving the full deduction of the gradient descent algorithm for training this type of networks. The performances of convolutional neural networks in the realvalued domain for image classification gave rise to the idea of extending them to the complex-valued domain, also. Real-valued image classification experiments done using the MNIST and CIFAR-10 datasets have shown an improvement in performance of complex-valued convolutional neural networks over their real-valued counterparts.

I. INTRODUCTION

Convolutional neural networks have become one of the most successful models in solving virtually any image recognition task. Proposed for the first time in [1], where they were used for handwritten digit recognition, they were later applied to handwriting recognition in [2]. By 1995, applications of this type of networks appeared in image recognition, speech recognition, and time series prediction [3]. Convolutional neural networks represent a particularization of feedforward neural networks, in which matrix multiplication is replaced by convolution and the weight matrix is replaced by many convolution kernels with much smaller dimension than that of the weight matrices. Although they had many applications in computer vision [4], convolutional networks started gaining more popularity only with the increase in the available computational resources and their implementation using parallel computing on graphics processors (GPU).

The use of these computational resources allowed a reducing of training times by a factor of 100, giving way to models with an increasing number of layers and parameters, thus inaugurating the domain of deep learning [5], [6], [7], [8]. The basis of this domain is represented by the convolutional networks, for which the increase in the number of layers gives better performance, as opposed to feedforward networks, whose performance degrades for a big number of layers. The same field includes other network models, for which it has been proved mathematically that performance is directly proportional to the size of the model.

The domain of complex-valued deep learning has appeared in the last few years. Although feedforward complex-valued neural networks have been applied to image recognition [9], and a single layer complex-valued convolutional neural network was used in [10] for object detection in Polarimetric Synthetic Aperture Radar (PolSAR) images, only in the last few years deep learning algorithms using complex numbers were derived and used. For example, in [11] complexvalued autoencoders are proposed, which are a type of model belonging to the deep learning paradigm. In [12] a wavelet scattering network is proposed, which uses complex numbers. Neuron synchrony in a complex-valued Deep Boltzmann Machine (DBM) was discussed in [13], showing superior performances to the real-valued case.

Very recent works discuss complex-valued recurrent neural network models [14], as well as learning time series representations using complex-valued recurrent networks [15], both with similar if not superior results than the real-valued ones, and the existence of certain properties of these networks that do not appear in the real-valued case, which makes them suitable for certain types of applications. One of the most important papers in this domain is [16], which gives a mathematical motivation for complex-valued convolutional neural networks, showing that they can be seen as nonlinear multiwavelet packets, thus making the mathematical analysis from the signal processing domain available for a rigorous formulation of the properties of complex-valued convolutional networks. Following the footsteps of this paper, it is expected that research in the complex-valued deep learning domain will increase in the coming years.

Taking the above discussion into consideration, a natural idea is to use complex-valued convolutional neural networks for image recognition, also taking into account the fact that some images are given by the imaging devices in complex form [17].

Synthetic Aperture Radars (SAR) are imaging systems that produce complex-valued images of the ground [18], [19], [20]. They can be Interferometric Synthetic Aperture Radars (InSAR) [21] or Polarimetric Synthetic Aperture Radars (PolSAR) [22], [23], [10]. Complex-valued neural networks were used for noise reduction, compression, and recognition of this type of complex-valued images. To date, to the best of our knowledge, there are only two attempts of recognition of this type of images directly in the complex domain, one using complex-valued feedforward neural networks, and one using a complex-valued convolutional neural network with a single layer, both models being more suitable to this problem than real-valued neural networks, which ignore the dependencies present in the data in the complex plane [23], [10]. This fact lead to the idea that deep convolutional neural networks could bring even better performance in this area.