

## TENSOR PRODUCT-BASED MODEL TRANSFORMATION USED IN CONTROL SYSTEMS MODELING AND DESIGN

### PhD Thesis – Summary

to obtain the scientific title of doctor at  
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The PhD thesis entitled „**Tensor Product-based Model Transformation Used in Control System Modeling and Design**” addresses the field of applying the Tensor Product-based Model Transformation technique (Tensor Product – TP) in modeling and control system design for three representative complex processes: the vertical three tank system, the magnetic levitation system and the pendulum cart system. The thesis is structured in five chapters and four appendices. A short overview of each chapter is presented as follows.

The scientific background along with a short general presentation of the thesis are presented in **Chapter 1**. The main objectives are also formulated.

The first objective of the thesis consists in the validation of the modeling algorithm of the TP-based Model Transformation technique on many laboratory equipments. The corresponding derived TP models are validated using many testing scenarios and they are compared with other models of the same processes in order to highlight their performance.

The second objective of the thesis consists in the validation of the control algorithm of the TP-based Model Transformation technique using Linear Matrix Inequalities (LMIs) and Parallel Distributed Compensation (PDC) framework. Therefore, many conventional and cascade control structures are designed for the control of various laboratory equipments. The proposed control structures are tested and compared with other similar ones and their performance is highlighted.

**Chapter 2** consists in a short general presentation of the main idea of TP-based Model Transformation technique and a bibliographic study which highlights the main theoretical and practical contributions obtained so far. Also, the main advantages and disadvantages of this technique are presented. The main advantage of the TP-based Model Transformation technique consists in the fact that it transforms Linear Parameter Varying (LPV) models into polytopic forms (Linear Time Invariants – LTIs) on which the LMIs techniques can be applied immediately. The main disadvantage of the TP-based Model Transformation technique consists in the large dimension of the core tensor of the derived TP model which generates: large computation volume, large execution time and large amount of memory.

In **Chapter 3**, the main steps of the TP-based Model Transformation modeling algorithm along with the derivation of TP models for three systems, namely Vertical Three Tank System (V3TS), partial state feedback controlled Magnetic Levitation System (psfMLS) and Pendulum Cart System (PCS) are presented.

In Sub-chapter 3.1, the steps of the TP-based Model Transformation modeling algorithm are described and details are given.

In Sub-chapter 3.2, the derivation of the TP model for a Vertical Three Tank System is presented. In order to carry out a comparative analysis, four linear models are also derived for

V3TS: the first two linear models are obtained by linearization around two operating points (o.p.s) and the next two linear models are extracted from the LTI system matrices of the TP model. Finally, the derived TP model is tested along with the nonlinear model of the V3TS, with four linear models and with the laboratory equipment using a Pseudo Random Binary Signal (PRBS) and four performance indices, namely Root Mean Square Error (RMSE), Value Accounted For (VAF), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are computed. The experimental results and the values of the performance indices show that the TP model ensures good modeling performance but exhibiting numerical error. The best performance concerning the values of RMSE is obtained by the fourth linear model in case of the first and second tank and by the TP model in case of the third tank while, the best performance in terms of VAF are obtained by the fourth linear model in case of all three tanks. However, the TP model ensures better performance than the nonlinear model and the four linear ones in terms of AIC and BIC in case of all three tanks.

In Sub-chapter 3.3, the derivation of the TP model for the **psfcMLS** is presented. In order to carry out a comparative analysis, four linear models are also derived for psfcMLS: the first two linear models are obtained by linearization around two o.p.s and the next two linear models are extracted from the LTI system matrices of the TP model. Finally, the derived TP model is tested along with the nonlinear model of the psfcMLS, with four linear models and with the laboratory equipment in the same four testing scenarios using PRBS, sine, chirp and Pulse Width Modulation (PWM) input signals. Also the four performance indices, namely RMSE, VAF, AIC and BIC are computed. The best performance concerning the values of RMSE is obtained by the TP model in the third testing scenario. The best performance concerning the values of VAF is obtained by the psfcMLS model in the first testing scenario. The third linear model ensures the best performance in terms of both AIC and BIC in case of the fourth testing scenario. The experimental results and the values of the performance indices, have shown that the TP model ensures good modeling performance but exhibiting nonzero numerical errors.

Sub-chapter 3.4 is dedicated to the derivation of the TP model for the PCS. In order to carry out a comparative analysis, four linear models are also derived for PCS: the first linear model is obtained by linearization around one o.p. and the next three linear models are extracted from the LTI system matrices of the TP model. Finally, the derived TP model is tested along with the nonlinear model of the PCS, with four linear models and with the laboratory equipment in the same two testing scenarios using sine and random input signals. Also, the four performance indices, namely RMSE, VAF, AIC and BIC are computed. The experimental results show that the derived TP model approximately mimics the behavior of the laboratory equipment, but exhibiting numerical error.

In Sub-chapter 3.5, the main contributions and the published papers are given.

In **Chapter 4**, the main steps of the TP-based Model Transformation control algorithm are given, along with the design of TP-based control structure (TPCS) for three systems, namely V3TS, psfcMLS and PCS. The TPCSs are compared with state feedback control structures (SFCSSs), which are designed aiming the same control design performance as in case of the TPCSs.

In Sub-chapter 4.1, the steps of the TP-based Model Transformation control algorithm are presented in detail.

In Sub-chapter 4.2, the validation through simulation and experiments of the TP controllers designed for V3TS is presented. At first the TPCS is designed. Then the TPCS is compared with four SFCSSs, which are designed aiming the same control performance as in case of the TPCS, and they are tested in the same scenario. Moreover, in order to improve the control system performance, i.e. to ensure zero steady-state control error, the TPCS and the four SFCSSs are included in 15 Single Input Single Output (SISO) Cascade Control Structures (CCSSs) designed for each of the three tanks with Proportional Integral Derivative (PID) controller in

the outer control loop, namely PID-TPCS and PID-SFCS. All control structures are tested in the same scenario and four performance indices, namely Mean Square Error (MSE), Mean Square Control Effort (MSU), settling time and overshoot are computed. The best performance concerning the MSE is achieved by the first PID-SFCS for the first tank in the simulation scenario and by the fourth PID-SFCS for the second tank in the experimental scenario. The best settling time is achieved by the third SFCS in case of all three tanks in the simulation scenario and by the four SFCSs for the third tank in the experimental scenario. The best performance in terms of MSU is obtained by the second SFCS in case of all three tanks in the simulation scenario and by the fourth PID-SFCS for the second tank in the experimental scenario. The overshoot is present in case of the PID-TPCS and the second PID-SFCS in the simulation scenario and in case of TPCS and the four SFCSs for the first and second tank and by the PID-TPCS, the second, the third and the fourth PID-SFCS for the first tank in the experimental scenario. Its smallest value is obtained for the PID-SFCS in case of the first two tanks, by the PID-TPCS in case of the third tank in the simulation scenario and by the TPCS and the four SFCSs for the second tank in the experimental scenario. The first five CSs, namely the TPCS and the SFCSs, do not ensure zero steady-state control error. Therefore, the implementation of the cascade control system structures is justified.

In Sub-chapter 4.3, the validation through simulation and experiments of the TP controllers designed for the psfMLS is presented. At first a TPCS is designed. The TPCS is next compared with four SFCSs, which are designed aiming the same control performance as in case of the TPCS. In the next step, in order to improve the control performance, i.e. to ensure zero steady state control error, the TPCS and the four SFCS are included in five SISO CCSs with a Proportional Integral (PI) controller in the outer control loop, namely PI-TPCS and PI-SFCS. The ten control structures, namely the TPCS, the four SFCSs, the PI-TPCS and the four PI-SFCS are tested in the same two scenarios (simulation and experiments) and the same performance indices as in case of V3TS are computed. In the simulation scenario, the best performance concerning the MSE is achieved by the second PI-SFCS while in the experimental scenario the best performance in terms of MSE is achieved by the PI-TPCS. The best performance in terms of MSU is obtained by the first SFCS in the simulation scenario and by the TPCS in the experimental scenario. The best settling time is achieved by all the four SFCSs in the simulation scenario and the settling time is similar for all control structures in the experimental scenario. The overshoot is present only in case of the PI-TPCS and of the first three SFCSs in the experimental scenario. The first five CSs, namely the TPCS and the SFCSs do not ensure zero steady-state control error in both testing scenarios. Therefore, the implementation of the cascade control system structures is again justified.

In Sub-chapter 4.4, the validation through simulations and experiments of the TP controllers designed for PCS in the crane operation mode is presented. At first a TPCS is designed. The TPCS is next compared with four SFCSs, which are designed aiming the same control performance as in case of the TPCS. In the next step, in order to improve the control performance, i.e. to ensure zero steady state control error, the TPCS and the four SFCS are included in five SISO CCSs with a Proportional Integral (PI) controller in the outer control loop, namely PI-TPCS and PI-SFCS. The ten control structures, namely the TPCS, the four SFCSs, the PI-TPCS and the four PI-SFCSs are tested in the same two scenarios (simulation and experiments) and the same performance indices as in case of PCS are computed. In the simulation scenario, the best performance concerning the MSE is achieved by the PI-TPCS while in the experimental scenario the best performance in terms of MSE is achieved by the first PI-SFCS. The best performance in terms of MSU is obtained by the third PI-SFCS in the simulation scenario and by the TPCS in the experimental scenario. The best settling time is achieved by the PI-TPCS in both the simulation scenario and the experimental. The overshoot is present only in case of the third and the fourth PI-SFCS in the simulation scenario and in case of the PI-TPCS and the four PI-SFCS in the experimental scenario. The first five CSs, namely

the TPCS and the SFCSs do not ensure zero steady-state control error in both testing scenarios. Therefore, the implementation of the cascade control system structures is once more justified.

In Sub-chapter 4.5, the main contributions and the published papers are given.

In **Chapter 5**, the main conclusions, the personal contributions, a list with the published papers and further research directions are presented. The results presented in this thesis are published in 14 papers. The author of the thesis is the first author of 12 out of the 14 published papers. The published papers are grouped based on the databases they are indexed in:

- 4 papers in journals with impact factor **indexed in Clarivate Analytics Web of Science** (with the former name ISI Web of Knowledge), with a **cumulative impact factor = 10.516** according to Journal Citation Reports (JCR) published by Clarivate Analytics in 2021; the author is the first author at **2 papers published in the journal Asian Journal of Control (with an impact factor = 3.452) in the Q2 quartile** and the first author for one of the other 2 papers published in journals in the Q3 quartile; one of the 2 papers published in Asian Journal of Control received the status „**Top Cited Article in 2020-2021**” according to Wiley
- 10 papers in conference proceedings indexed in Clarivate Analytics Web of Science (with the former name ISI Web of Knowledge); among these conferences there are some **main conferences of the IEEE societies (International Conference on Systems, Man and Cybernetics – IEEE SMC, International Symposium on Industrial Electronics – ISIE)** and the representative control conference in Romania (**International Conference on System Theory, Control and Computing**).

The published papers received a total number of **49 independent citations** (excluding the self-citations and the citations of all the co-authors) with a **cumulative impact factor = 150.922**. The citations are grouped by the database in which they are indexed:

- 44 citations indexed in Clarivate Analytics Web of Science (43 in journals and 1 in proceedings);
- 5 citations indexed in Google Scholar.

The thesis has four appendices where the numerical values of the parameters of the TP models and the TP controllers are given.

The PhD thesis contains:

- 116 pages,
- 79 figures,
- 19 tables and
- 122 state-of-the-art references.

### Significant references

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