Contributions regarding the development of cellular structures destined for sports equipment

Ph.D. Thesis – Abstract

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author Prof. Sorin Octavian SĂRĂNDAN
scientific coordinator Prof.Ph.D.Eng. Liviu MARŞAVINA

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The doctoral thesis focuses on developing new materials based on cellular structures, called metamaterials, and recommends their use in the design and manufacture of sports equipment, particularly the soles of sport shoes. The thesis is structured in six chapters, presented in this abstract.

1. Introduction

In the first part of the introduction, the choice of this research topic is justified. In the last decades, professional sports have become a real industry supported financially by significant marketing programs.

The unique growth of the sports industry led to the development of manufacturing companies for sports equipment. Therefore, the profits of these companies have increased due to the emergence of specialized retailers, the accessibility of products supported by the surge in living standards, the association of sports equipment brands with top professional athletes, etc.

In this context, the exceptional financial benefits have allowed the manufacturing companies to allocate substantial budgets to the research and development of new products. The main aspects pursued by research and development activities are the following: reducing the mass of the equipment, respectively customizing their elastic and mechanical characteristics.

In this trend, the subject of the study consists, on the one hand, in the development of active components with reduced densities, which will have comparable elastic and mechanical characteristics to those made of conventional materials, and on the other hand, in the evaluation of the possibility of individual design and manufacture, considering the determination of the performance for the product.

In the second part of the introduction, the bibliographic study establishes the main characteristics of sports equipment: protection of certain areas of the body, improvement of the individual performance and the assurance of stability and comfort of the athletes [1]. The tendency is to integrate new materials in the manufacture of sports equipment, like EVA, PVC, polyurethane, composite materials, and various combinations of them [2, 3]. Also, diversification based on types of sports and classes of exercises represents another research direction [4].

A distinct category of materials used in the sports equipment industry is represented by cellular materials, especially polymeric foams, with good damping properties and the absorption of kinetic energy upon impact, its storage and partial return [5]. The year 2017 sets a milestone for these materials, as the first model of sports shoes made of printed cellular structures appeared, the Adidas Futurecraft 4D. Shortly after that, macrostructures that used arrangement of cells and their density for the best configuration emerged and got recognition as good opportunities to improve performances of sports shoes. The Enerzy Mizuno model introduces the first composite cellular foam, EVA combined with a central rubber area. Adidas recently introduced boost foams, representing expanded thermoplastic polyurethane granules around small air bubbles, obtaining closed-walled cells.

The objectives of the research are stated in the last part of the introduction, together with the steps to be taken to achieve these objectives. The doctoral thesis aims to develop new sports equipment based on cellular structures that, on the one hand, have superior properties compared to conventional equipment (either increased mechanical properties for the same mass, or reduced masses for similar performances), and, on the other hand, to be easily modified according to the preferences and needs of each athlete. In order to achieve this goal, a series of steps were considered: investigation of the mechanical properties of the cellular materials used in the design of sports equipment, as well as those of the materials that will be the source of the designed structures; calibrating the constituent models in order to reproduce as accurately as possible the particularities of this class of materials; investigation of several types of metamaterial structures, for the selection of those that will be implemented; modeling of selected structures in order to integrate them into sports equipment; determining the feasibility of sports equipment made of metamaterial structures based on the results of numerical analyzes performed on them and comparing with the results obtained on reference model, which are composed of conventional materials.

2. Mechanical characterization of the materials used for sports equipment

Considering the characteristics of flexible foams and elastomers, an exhaustive characterization included monotonic tests (tensile and compression), relaxation tests, and cyclic load tests.

The tests were performed on two types of specimens, cylindrical compression specimens and *dogbone* type specimens intended for tensile tests. Two categories of materials were tested: specimens were extracted from five different models of sports shoes, and specimens fabricated by rapid prototyping from TPU and Agil-U materials (figure 1).

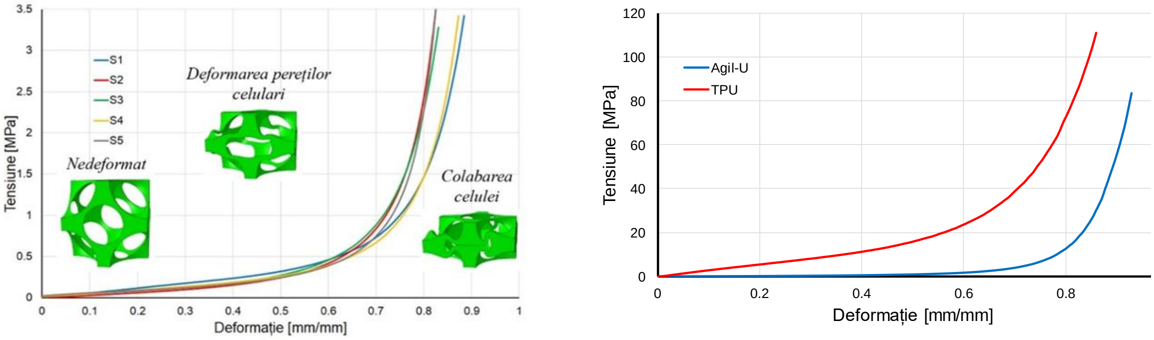
(a) (b)

Fig. 1. Dogbone specimens (a) taken from sports shoes (b) realized through rapid prototyping

The compression tests were conducted under the following conditions: crosshead travel speed of 10 mm/min at ambient temperature. Materials extracted from the soles of sports shoes showed a similar non-linear behavior, having two distinct regions: a more compliant one, determined by cell walls deformation, and a stiffer region, characterized by cells collapse. At the specific strain of 80%, the average value of the mechanical strength reached 1.93 MPa. The TPU elastomer presented a significantly higher stiffness than that determined for Agil-U (figure 2). The mechanical strengths obtained for the two elastomers are significantly higher than the values obtained for conventional materials at a similar specific strain.

The static tensile tests were carried out under the following conditions: crosshead speed of 10 mm/min at ambient temperature. The strain of the samples was determined using the *mark-tracking* method. The stress-strain curves for the materials of the soles of sports shoes exhibit a slightly different behavior for them, with a greater spread for the results (figure 3).

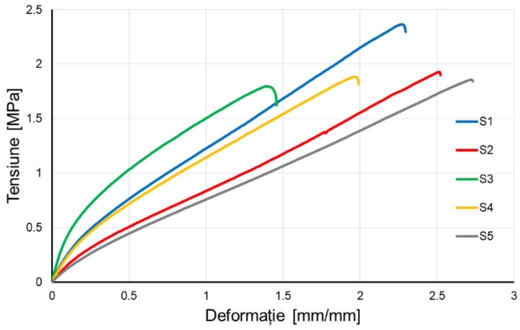
The average mechanical strength reached 1.96 MPa, with an average specific strain at break equal to 216.9%.



(a) (b)

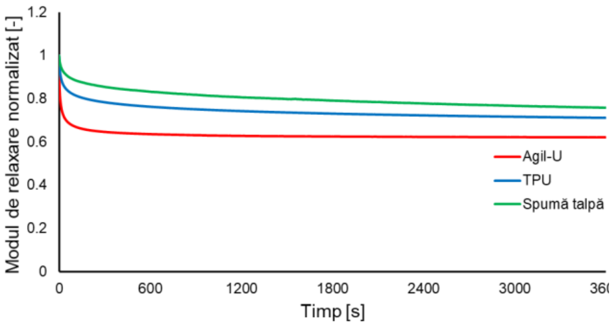
Fig.2. Stress-strain curves for compression tests (a) specimens extracted from sports shoes (b) elastomers realized through rapid prototyping

Tests performed on TPU presented a mechanical strength equal to 28.21 MPa, with a higher strain, the latter explained by the used printing direction. The mechanical strength of Agil-U material is approximately two times lower, reaching 0.80 MPa, but the strain is of the same order of magnitude as that determined for the materials taken from the soles of sports shoes.



(a)

Fig.3. σ-ε curves for tensile tests (a) specimens extracted from sports shoes (b) elastomers fabricated by rapid prototyping



(b)

Fig.4. Variation of the normalized relaxation modulus e(t) for the investigated materials

Stress relaxation tests were performed by applying a compression load, for a strain of 25%, maintained for 60 minutes. Figure 4 shows the variations of the normalized relaxation modulus e(t) for the studied materials. Their behavior is similar, with Agil-U elastomer showing the lowest normalized relaxation modulus and the sports shoe sole sample, the highest value [6].

The cyclic tests were conducted with a compression load applied for three different levels, defined by the specific strain equal to 25%, 50% and 75%. For each one, 50 load cycles were performed at a frequency of 0.5 Hz. The stress-strain curves described by the monotonic tests are tangent to the curves described by the cyclic loads. Viscoelastic effects are more pronounced at relatively low values of the applied load.

Analyzing the results of the cyclic tests, it was concluded that the investigated materials exhibit the three characteristics specific to the Mullins effect: a gradual decrease in stiffness caused by multiple cyclic loads in the same stress ranges, the resuming of the path described by a monotonic test after exceeding the maximum value of the stress determined in previous cycles and a hysteresis loop described during unloading [7, 8].

3. **Calibration of visco-hyperelastic material models**

The mathematical models which were used in simulating the behavior of hyperelastic materials and flexible foams were presented in the first part of this chapter. Although they show similar characteristics during deformation, the key difference between these two classes of materials is given by the variation of volume during deformation. Hyperelastic materials, such as rubber, show very small volume deformations while tested (being considered incompressible in some applications), while flexible foams show very large volume deformations during testing, having a Poisson's ratio with very small values (close to 0 for certain types of flexible foams).

Unlike conventional materials (metals, ceramics and some polymers), which show a linear stress-strain curve in the first region of the chart (behavior being modeled by Hooke's law), the analysis of the experimental results uncovered the fact that hyperelastic materials and flexible foams exhibit a non-linear characteristic curve. For these types of materials, modeling the mechanical behavior needs a different approach, involving the strain energy (elastic potential of the material). Therefore, the stress determined by a certain strain is obtained by deriving the specific energy with respect to the strain.

The specific energy's variation with the strain is determined from the experimental data, by integrating the stress values with the strain. Over the years, various mathematical models have been developed (also called hyperelastic functions) to express the variation of specific energy with strain or with other conventional quantities used in quantifying the change in dimensions for solids (such as specific extensions). To calibrate these models means to determine the material constants used in these functions, based on experimental data, so that the analytical variation of specific energy with strain is as close as possible to the experimental variation. As examples of such models for hyperelastic materials, the polynomial function (equation 3.1) and the Ogden function (equation 3.2) could be mentioned. For flexible foams, the model used to simulate the behavior is the Ogden function, modified with an element to take into account deformations in volume (equation 3.3), [9].

$$\psi = \sum_{i+j=1}^{n} C_{ij}(I_\lambda - 3)^i (II_\lambda - 3)^j + \sum_{k=1}^{m} \frac{1}{D_k}(J-1)^{2k} \tag{3.1}$$

$$\psi = \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i^2}\left[\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right] \tag{3.2}$$

$$\psi = \sum_{i=1}^{n} \frac{2\mu_i}{\alpha_i^2}\left\{\hat{\lambda}_1^{\alpha_i} + \hat{\lambda}_2^{\alpha_i} + \hat{\lambda}_3^{\alpha_i} - 3 + \frac{1}{\beta_i}\left[(J^{el})^{-\alpha_i\beta_i} - 1\right]\right\} \tag{3.3}$$

Defining the behavior of these materials to cyclic loads implies the reduction of the instantaneous elastic response of the material on the unloading region, thus generating a hysteresis phenomenon. One of the most used formulations is Ogden-Roxburgh, which involves multiplying the specific strain energy by a factor that takes into account the maximum value of the specific energy, recorded during previous tests, and three material constants (equation 3.4), [10].

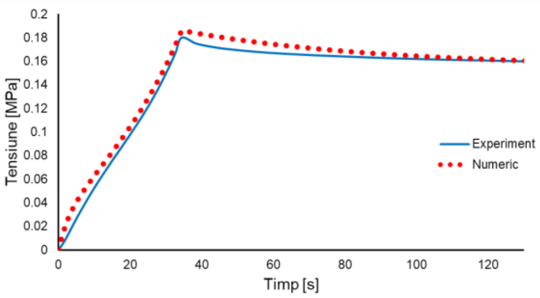
$$\begin{cases} \psi = \eta\Psi, \;\; \eta \in (0,1] \\ \eta(\Psi) = 1 - \frac{1}{r}\operatorname{erf}\left(\frac{\Psi^{max} - \Psi}{m + \beta \cdot \Psi^{max}}\right) \end{cases} \tag{3.4}$$

The long-term effect of loads on the mechanical behavior of materials can be modeled by numerous approaches, using Boltzmann's principle (superposition principle), creep laws or rheological models. The model used in this research was based on the generalized Kelvin-Voigt rheological model, expressed mathematically with the help of Prony series (equation 3.5). This model expresses the normalized modulus of elasticity as a function of time and some material constants. The principle of modeling viscoelastic phenomena is similar to that used in cyclic tests, namely, during loads, the instantaneous elastic response of the material is reduced by a dimensionless coefficient determined by the Prony series, [11].

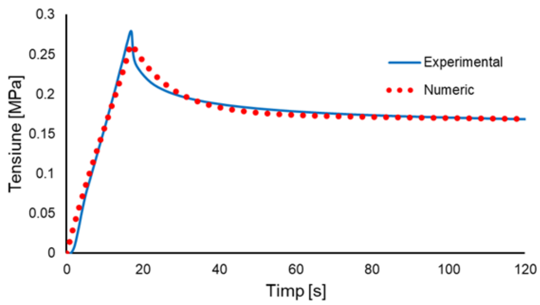
$$e(t) = 1 - \sum_{i=1}^{N} e_i\left(1 - e^{-t/\tau_i}\right) \tag{3.5}$$

Calibration for the models of materials involved an iterative process, due to the fact that the calibration of the hyperelastic functions required the instantaneous theoretical response of the material (the stress-strain curve determined by an infinite strain rate). The first step was to evaluate the hyperelastic models from experimental data obtained for compression tests at relatively low strain rates, and the relaxation tests were used in the calibration of the Prony series. Due to the viscoelastic effects caused by the rheological model, the numerical analyzes for compressive loads, using previously formulations, determined lower stress values for the same strains, compared to the experimental results. Accordingly, in the second iteration, the coefficients of the hyperelastic functions were determined based on the instantaneous theoretical response of the materials, estimated from multiplying the experimentally obtained stress values with a factor determined from the ratio of the experimental stresses and the values obtained from the numerical analyses.

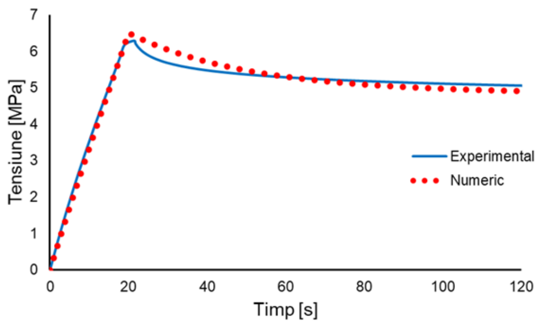
The second part of the chapter described the calibration procedures for the material models used in the simulation of flexible foams for sports shoe soles. The final model was evaluated by replicating the relaxation tests, which included a loading region and a strain-holding region, obtaining good results (Fig. 5a).



a)



b)



c)

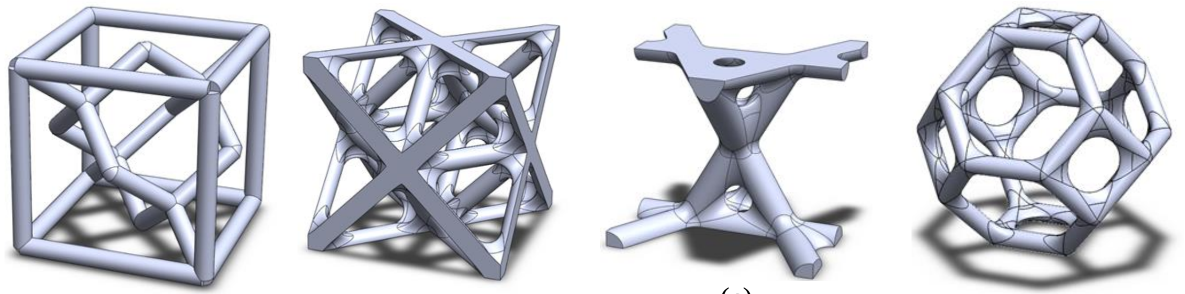
Fig. 5. Comparison between numerical and experimental relaxation test results for flexible foams (a), Agil-U (b), respectively TPU (c).

In the third part of the chapter, calibration methods for elastomers obtained through additive manufacturing were described, using the same procedures previously explained. Also, for these materials, the results, obtained after evaluation of the material models for the relaxation tests, determined accurate results (Figure 5b and 5c).

4. Metamaterial structures for sports applications

Metamaterials are considered periodic structures (tessellations) with well-defined geometries, which have superior properties than foams with similar densities [12, 13]. The metamaterial structures are grouped into two categories, structures found in nature, respectively anthropogenic structures [14-16]. The first category includes structures with cubic cells based on crystal lattices (face-centered cubic structure, volume-centered cubic structure, diamond's crystal structure), and from the second category the best known are Kagome, Kelvin and octet-beam structures.

For sports equipment applications, four types of structures were selected to be investigated: the structure based on the diamond cubic crystal system, the octet-beam structure, the Kagome structure and the Kelvin structure (Figure 6). The representative volume element was composed of 2×2×2 cells, except for the Kagome structure where 3×4×2 cells were joined.



(a) (b) (c) (d)

Fig.6. The cells of the investigated metamaterial structures (a) structure based on the diamond cubic crystal system (b) octet-beam structure (c) Kagome structure (d) Kelvin structure

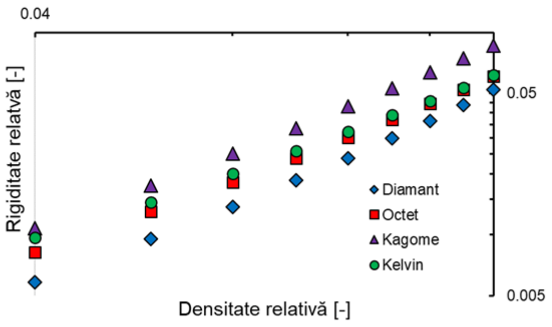
Firstly, geometric models for the four structures were developed, including connections between beams, with the aim of reducing the stress concentration phenomenon [17]. The characteristic dimensions (beam diameter d, beam length l and the connection radius) were parameterized, allowing different relative densities to be obtained by changing values.

In the second step, the variation of the relative density of the structures with the d/l ratio was determined. The influence of the joint radius was discarded by considering the maximum radius that determined a valid geometry. The variation of the relative density with the d/l parameter was expressed with the help of the polynomial function of the form $\rho_{rel} = f(d/l)$, the ratio between the diameter and the length of the beams being obtained for predetermined values of the relative density, such as $\rho_{rel} = 0.04; 0.06; 0.08 \dots 0.18; 0.2$. To determine the variation of the connection radius and the relative density with the ratio d/l, the length of the beam was kept constant, and for the diameter were used preset values.

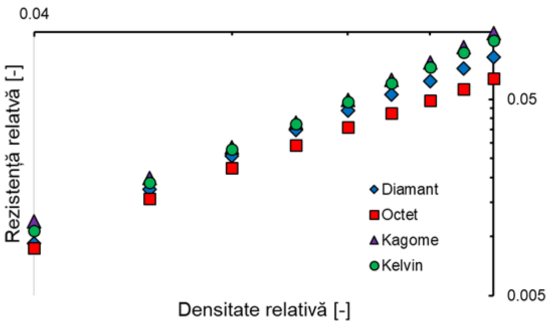
In the third step, numerical analyzes were performed on the structures in the Abaqus/CAE software package for compressive loads.

In the numerical analyses, a linear-elastic and a plasticity formulation characterized by isotropic multi-linear hardening were used for the material. The meshing used second order C3D10 tetrahedral elements with a minimum of three finite elements per beam section. Periodical boundary conditions were applied to the structures, with the compression load considered by applying a displacement. From the displacements and reactions of the recorded nodes, the nominal strains and stresses were calculated.

Finally, the variation of the relative stiffness, respectively the relative strength, with the relative density (Figure 7) and the stress-strain curves (Figure 8) were obtained.



(a)



(b)

Fig.7. Variation with relative density of relative stiffness (a) and relative strength (b)

With superior properties and higher energy absorption capacity, the Kagome and Kelvin structures were selected for implementation in sports equipment.

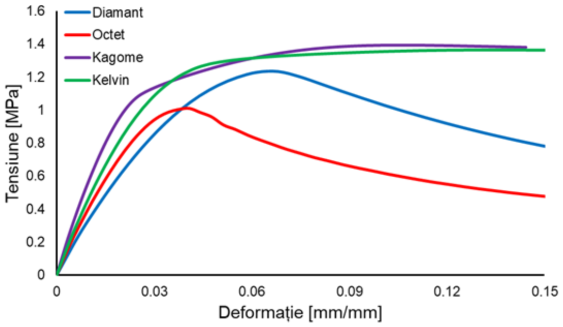


Fig. 8. Stress-strain curves of the four structures for relative density equal to 0.12

5. Implementation of new materials in sports equipment

To evaluate the performances of the new geometric models of sports shoe soles, in the first step, a reference analysis was performed on a sole with similar properties to those of the foams investigated in the second chapter. The sole model was downloaded from the GrabCAD platform (Figure 9).

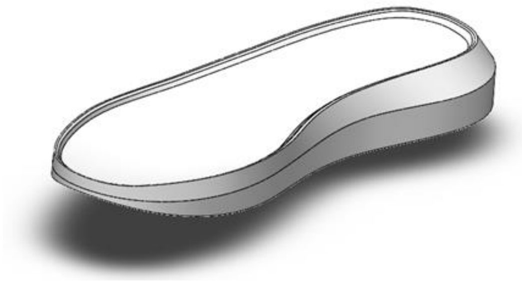


Fig.9. CAD model of sports shoe sole

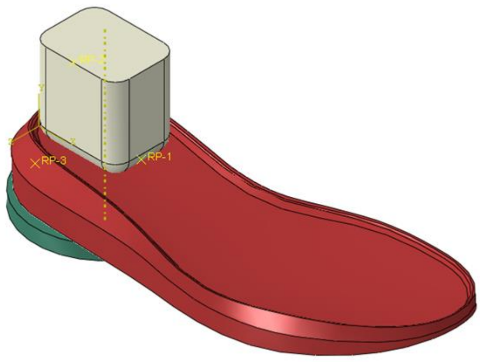


Fig.10. CAD model of sports shoe sole

The assembly considered in the numerical analysis consists of a rigid indenter, a rigid support and the sole. The role of the indenter is to simulate the human heel during walking, and the support to simulate contact with the ground, Figure 10, [18].

The model of material used for the sole consisted of a mathematical formulation for flexible foams (*Hyperfoam*), a relaxation model and a model for the Mullins effect.

The meshing of the indenter and the support utilized rigid finite elements of type R3D4, using a free meshing technique. For the sole, the meshing was performed with second order C3D10 finite elements.

The contact between the components of the assembly required a normal formulation of the "hard contact" type and a tangential formulation, with a friction coefficient of 0.2.

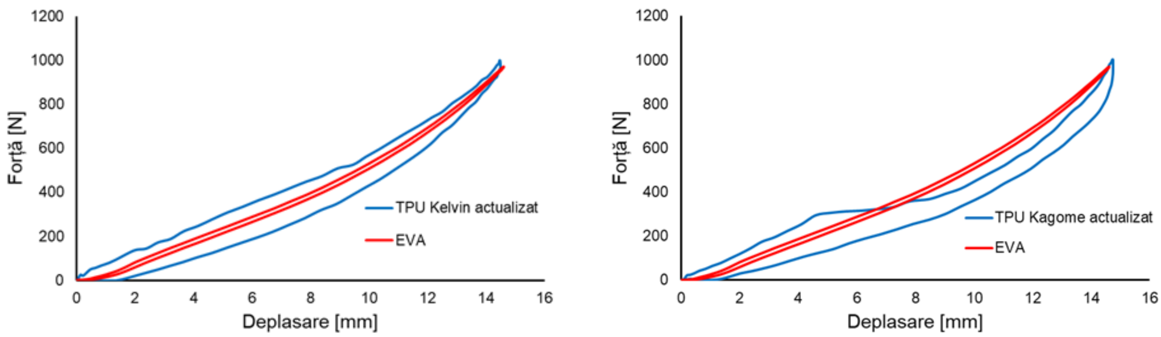
Boundary conditions consisted of fixing the indenter and assigning a vertical displacement for the rigid plate following a triangular amplitude to simulate loading-unloading cycle. The condition to reverse the direction of movement for the support was the reach a reaction force value of 1000 (N).

The data extracted from the numerical analyses are: stress and strain field, and force-displacement curve. The results are consistent with those reported in technical literature [19].

For the sports shoe soles designed using Kagome and Kelvin structures, numerical analyzes with the finite element method were performed under similar conditions (same interactions, boundary conditions, meshing of rigid components, etc.). Material properties which are characteristic of TPU thermoplastic polyurethane have been assigned. The geometric model of the sole was generated using a negative of it, through a Boolean operation of removing the regions from the metamaterial structure that encounter the negative of the model.

The initial model based on the metamaterial structures was designed in a manner that makes its mass equal to that of the reference model (using EVA as assigned material). The models generated from the metamaterial structures determined higher stiffnesses, with a certain factor *k*, specific to each structure.

To obtain comparable results to the reference model, a reduction of the elastic properties of the soles based on metamaterial structures was imposed. Thus, for the relative stiffness decreased by the factor *k*, the relative density was calculated at the imposed value of the relative stiffness, and finally the value of the geometric ratio d/l corresponding to the updated structure was determined. The geometric models obtained had reduced mass, 50.7 gr. for the Kelvin structure and 30.16 gr. for the Kagome structure, by comparison with the reference model (EVA), whose mass was equal to 94.2 gr.



(a) (b)

Fig.11. Comparison of the force-displacement curves for the reference model (EVA) and the Kelvin structure-based model (a) and the Kagome structure-based model (b)

From figure 11a we can see that, although they have a similar slope during loading, the sole based on the Kelvin structure causes greater reactions for the same deformations, that is because the sole made of EVA foam presents a settling region in the beginning. At the same time, the sole based on the Kelvin structure exhibits a more significant hysteresis loop, with viscoelastic effects causing full elastic recovery (zero reaction) at a displacement of approximately 1.5 mm.

Figure 11b contains the comparison between the force-displacement curve of the reference model (EVA) and that of the updated model generated from the Kagome structure. The behavior of this sole model shows similarities with the updated sole model based on the Kelvin structure. Even though in the first section the slope of the curve determined by the reference model and that of the model based on the Kagome structure are similar, the latter causes larger reactions for the same displacement value. The observation is explained by the settling region of the reference model. A much more pronounced plateau region appears in the case of the sole based on the Kagome structure, this behavior being determined by the buckling of the beams.

The last region of the force-displacement curve indicates a progressive increase in the stiffness of the structure, explained by the contact of the beams, a step which is considered as the beginning of the densification region.

The viscoelastic effects are similar for the two types of structures, the hysteresis curves showing similar shapes. The complete recovery of both structures, corresponding to the zero value of the reaction at unloading, is recorded when a displacement of 1.5 mm is reached.

It can be concluded that, although it has a larger mass, the Kelvin-type structure offers better results than the Kagome-type structure due to the uniformity of the reaction to the compressive loads. Thus, the behavior of the sole generated on the Kelvin structure is much more predictable while used. For the sole generated from the Kagome structure, the sudden change in stiffness could cause discomfort to the athlete.

6. Conclusions and personal contributions

The doctoral thesis brings original elements to the investigated field, the manufacture of sports equipment based on advanced materials.

Within the experimental program, the author's main contributions relate to the determination of the mechanical behavior of distinct types of cellular materials used in the manufacture of sports shoes and protective equipment. In addition to highlighting the characteristics determined in uniaxial monotonic compression and tensile tests, defining the cyclic and long-term loads behavior for these types of materials is an important contribution. At the same time, determination of the mechanical properties of elastomers obtained through additive manufacture is a subject of interest, taking into account the increasing proportion of applications of components manufactured by these technologies.

Calibration of virtual material models highlights several personal contributions, namely: overlaying various constitutive models, hyper-elasticity/flexible foams with Prony series and Ogden-Roxburgh model; replicating monotonous, cyclical and long-term loads with highly accurate results.

The selection of the optimal structures was based on the analysis of structures described in literature. The main personal contributions consist in determination of the variation of the maximum connection radii with the dimensional parameters, respectively the variation of the relative density with the geometrical parameters. Another novelty that the author contributed with is the numerical analysis of the structures using an elastic-plastic formulation of the base materials. The variation of relative stiffness and relative mechanical strength with relative densities was obtained, considering the complex states that occur as a result of plastic deformations.

The main contribution of the study is the design of components based on metamaterial structures and the numerical evaluation of the mechanical behavior for loads observed during use. The analysis carried out highlighted the superiority of the new components compared to the conventional ones, resulting in development opportunities for new products.

Bibliografie

[1] T.G. McPoil, “Athletic Footwear: design, performance and selection issue”, *Journal of Science and Medicine in Sport*, 3 (3), pp. 260-267, 2000.
[2] J. Indesteege, R.E. Camargo, P.W. Mackey, G. Bagaglio, A. Dent, “Considerations on the selection of polyurethane systems for midsole wedges in athletic footwear application”, *Journal of Cellular Plastics*, 34 (4), pp. 329-348, 1998.
[3] Y. Shimazaki, S. Nozu, T. Inoue “Shock-absorption properties of functionally graded EVA laminates for footwear design”, *Polymer Testing*, 54, pp. 98-103, 2016.
[4] S.I. Subotnick, C. King, M. Vartivarian, C. Klaisri “Evolution of Athletic Footwear”, pp. 3-17, in: *Athletic Footwear and Orthoses in Sports Medicine*, Ed. M.B. Werd, E.L. Knight, Springer, 2010.
[5] L. Gibson, M. Ashby, *Cellular Solids. Structure and Properties*, Cambridge University Press 2nd ed., UK, 1997.
[6] S. Boedicker, S. Hunter, P. Klesmith, T. Nearn, P. Spreen “Mechanical properties testing of running shoes”, *Journal of Undergraduate Materials Research*, 4 (1), pp. 34-38, 2010.
[7] S. Cantournet, D. Desmorat, J. Besson, “Mullins effect and cyclic stress softening of filled elastomers by internal sliding and friction thermodynamics model”, *International Journal of Solids and Structures*, 46, pp. 2255-2264, 2009.
[8] A. Drozdov, N. Düşünceli, “Mullins-type phenomena in polypropilene”, *International Journal of Applied Mathematics and Mechanics*, 8 (11), pp. 82-98, 2012.
[9] H. Brinson, L. Brinson, *Polymer Engineering Science and Viscoelasticity: An Introduction*, Springer Science, 2008.
[10] R.W. Ogden, D,G. Roxburgh, “A pseudo-elastic model for the Mullins effect in filled rubber”, *Proceedings of the Royal Society of London A*, 455, pp. 2861-2877, 1988.
[11] D. Bower, *An Introduction to Polymer Physics*, Cambridge University Press, 2002.
[12] X. Zheng, H. Lee, T.H. Weisgraber, M. Shusteff, J. DeOtte, E.B. Duoss, J.D. Kuntz, M. M. Biener, Q. Ge, J.A. Jackson, S.O. Kucheyev, N.X. Fang, C.M. Spadaccini “Ultralight, ultrastiff mechanical metamaterials”, *Science*, 344 (6190), pp. 1373-1377, 2014.
[13] R.S: Kshetrimayum, “A brief intro to metamaterials”, *IEEE Potentials*, 23 (5), pp. 44-46, 2004.
[14] M. Eidini, “Zigzag-base folded sheet cellular mechanical metamaterials”, *Extreme Mechanics Letters*, 6, pp. 96-102, 2016.
[15] N. Engheta, R.W. Ziolkowski, *Metamaterials: Physics and Engineering Explorations*, Wiley-IEEE Press, 2006.
[16] K. Wang, Y.H. Chang, Y.W. Chen, C. Zhang, B. Wang “Designable dual-material auxetic metamaterials using three-dimensional printing”, *Materials and Design*, 67, pp. 159-164, 2015.
[17] D. Șerban, R. Negru, S. Sărăndan, G. Belgiu, L. Marșavina “Numerical and experimental investigations on the mechanical properties of cellular structures with open Kelvin cells”, *Mechanics of Advanced Materials and Structures*, 28 (13), pp. 1367-1376, 2021.
[18] S. Schwanitz, S. Möser, S. Odenwald, “Comparison of test methods to quantify shock attenuating properties of athletic footwear”, *Procedia Engineering*, 2, pp. 2805-2810, 2010.
[19] R. Verdejo, N. Mills, “Performance of EVA foam in running shoes”, in: S. Ujihashi, S.J. Haake (eds.), *The Engineering of Sport 4*, pp. 580-587, Blackwell, 2002.