# BULETINUL ȘTIINȚIFIC al Universității "Politehnica" din Timișoara, Romania SCIENTIFIC BULLETIN OF "Politehnica" University of Timișoara, Romania

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### AN ALTERNATIVE TO LAGRANGE METHOD OF VARIATION OF CONSTANTS

# Miloje RAJOVIĆ, Milena LEKIĆ and Dragan DIMITROVSKI

**Abstract.** In this paper an analogue to Lagrange method of variation of constants is proposed. Beside the compulsory solving of appropriate homogeneous equation, instead of solving a system of linear differential equations in regard to constants  $C_1, C_2, ..., C_n$  regarding functions, a linear differential equation of an order less by at least one is being solved. Basic theorems on existence of solution of the problem are given. Furthermore, important application possibilities are shown, for example with resonance.

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### SEQUENCES OF REAL NUMBERS AND DIAGONALIZATION PROCESSES

### Ljubiša D.R. KOČINAC, Dragan DJURČIĆ and Mališa ŽIŽOVIĆ

**Abstract.** We review some results concerning investigation of diagonalization processes in some classes of sequences of positive real numbers.

Keywords and phrases: Selection principles, game theory, Ramsey theory, regular variation, rapid variability.

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#### ABOUT APPROXIMATION OF B-CONTINUOUS AND B-DIFFERENTIAL FUNCTIONS OF THREE VARIABLES BY GBS OPERATORS OF BERNSTEIN-SCHURER TYPE

#### Mircea D. FARCAŞ

**Abstract.** In this article, using a method from the paper [4], the sequence of GBS operators of Bernstein-Schurer type for functions of three variables is constructed and some approximation properties of this sequence are established.

*Keywords and phrases*: Bernstein-Schurer operators, GBS operators, functions of three variables, B-continuous, B-differentiable, mixed modulus of smoothness.

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### SOME DEVELOPMENTS OF FUNDAMENTAL PARATRIGONOMETRIC EQUATION

# Malvina BAICA and Mircea CÂRDU

Abstract. In this paper the authors extend the validity of the paratrigonometric fundamental equations which apply for the under unit values of x to its uperunit values. Also, some special algebraic equations are established.

Keywords and phrases: Paratrigonometry, Basic trigonometric figures, Essential equations.

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#### EINSTEIN METRICS VIA HOLONOMY AND CONE FIELDS

### Cătălin C. VASII

**Abstract.** This paper should be regarded as a following of [Vasii], but could read independently. Results from [Vasii] are listed. Here, using cone fiends on vector bundle, the author defines a notion of positive parallel transport, via a correspondent notion of a curves lift, and gives a description of Einstein metrics in this language. Then a notion of positive holonomy in introduced, this again helps characterizing Einstein metrics, again with a specific notion of harmonicity.

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### SOME REMARKS ABOUT NUMERICAL INTEGRATION OF A SYSTEM OF DIFFERENTIAL EQUATIONS ON THE HEISENBERG LIE GROUP H(4) (II)

#### Anania ARON and Camelia POP

**Abstract.** The goal of our paper is to make a comparison between two numerical integrators on the Lie group H(4) and to point out their properties.

*Keywords*: optimal control, Lie-Trotter algorithm, Kahan algorithm, Runge-Kutta 4<sup>th</sup> stages algorithm.

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# SOME REMARKS ON AN OPTIMAL CONTROL PROBLEM ON <sup>3</sup>

### **Ciprian HEDREA**

Abstract. An optimal control problem on  $^{-3}$  is discussed and some of its properties are pointed out.

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#### GEOMETRICAL AND DYNAMICAL ASPECTS ON THE LIE GROUP SE(2, R) × SO(2)

#### **Camelia POP and Anania ARON**

**Abstract.** The goal of our paper is to present an optimal control problem on the Lie group  $SE(2, R) \times SO(2)$  and to point out some of its geometrical and dynamical properties.

Keywords: optimal control, drift-free controllable system

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### ON SOLVING THE RICCATI EQUATION AND ITS EQUIVALENCE WITH A SECOND ORDER LINEAR EQUATION

#### **Cristinel MORTICI**

**Abstract.** We will establish here the equivalence of the Riccati equation with a second order linear differential equation. The particular solution of the Riccati equation produces a particular solution for the corresponding linear equation. By using the Liouville theorem about the wronskian, we reduce the order of the linear equation, then we can completely solve it. In consequence, an explicit formula for solving the Riccati equation follows. The equivalence between Riccati equation and a second order linear equation can be start point for developing other nice results in the theory of the ordinary differential equations.

Keywords: linear equation, Riccati equation

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**Physics** 

## FINITE DIFFERENCE SOLUTION FOR THE 1-DIMENSIONAL QUANTUM HYDRODYNAMIC MODEL EQUATIONS

# Bogdan CARUNTU, Ioan ZAHARIE, Romeo NEGREA, Viorel CHIRITOIU

**Abstract:** We present a numerical study of the 1-dimensional quantum hydrodynamic model equations describing the behavior of the electrons in a thin layer of a photovoltaic cell when the semiconductor is uniformly doped.

For an associated initial value problem, first an explicit finite difference scheme is proposed, based on the scheme employed by Zabusky and Kruskal [6] in the study of the Korteg de Vries equations.

In order to overcome some limitations with respect to the convergence and accuracy of the explicit scheme, we also proposed an implicit finite difference scheme with better convergence properties.

*Keywords:* quantum hydrodynamic model, semiconductor, photovoltaic cell

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# NUMERICAL STUDY OF QUANTUM HYDRODYNAMIC MODEL FOR SEMICONDUCTORS

#### Viorel CHIRIȚOIU, Ioan ZAHARIE, Ioan LUMINOSU

Abstract: This paper presents a numerical study of the one dimensional quantum hydrodynamic equations, introducing the quantum hydrodynamic model (QHD) for semiconductors. In the case of QHD, the numerical solution of Schrödinger equation must present higher oscillations, as the scaled Planck constant  $\varepsilon$  becomes smaller ( $\varepsilon \sim 10^{-3} \div 10^{-2}$ ). The numerical studies for general case and for particular isothermal, stationary case are given. Finally we present different graphical solutions for particle and current densities, in both cases and for different values of  $\varepsilon$ . From graphical representations it can be observed an increasing amplitude of solutions' oscillations of particle and current densities as  $\varepsilon$  becomes smaller. For stationary case it can be observed that current density remains constant indifferent of  $\varepsilon$ 's choice.

Keywords: quantum hydrodynamic model, semiconductor, photovoltaic cell

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# IR TRANSMITTANCE SPECTRA IN THE TERNARY OXIDIC Me<sub>2</sub>O<sub>3</sub> SYSTEM

### Minerva CRISTEA, Marius COSTACHE, Ioan LUMINOSU, Viorel CHIRITOIU

**Abstract**. The experimental results concerning the infrared transmittance spectra (400-4000 cm<sup>-1</sup>) of samples with 70% mol  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> from the isostructural oxides  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Al<sub>2</sub>O<sub>3</sub> ternary system are presented and discussed. The IR spectra were obtained by using powdered samples tablet formed with KBr, at room temperature. Some investigated samples are solid solutions and the others contain two phases and these belong to gap miscibility from the ternary system. All spectra exhibit two absorption bands below 700 cm<sup>-1</sup> that are larger for samples from gap miscibility.

Keywords: IR spectra, oxydic solid solution, sescquioxides structure

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# THE VIBRATIONAL ENERGY TRANSFER IN HOMOGENEOUS AND ISOTROPIC HEAT FIELD-COMPENSATION EFFECT

Nicolina POP, Nicolae DOCA

**Abstract:** The topic of this work is to suggest an activation mechanism who lead to dependences similar with the compensation effect (CE).

Considering an activation process due to a vibrational energy transfer from an homogeneous and isotropic field of thermic oscillators to the solid state oscillator, the thermodynamic functions are in the relationship:  $dH = T \cdot dS$ . This is in fact equivalent with one of the mathematical expression of CE.

An important consequence, a correlation between the isokinetic temperature and the spectroscopic wave number of the activated bond, is illustrated by a number of decomposition reactions under non-isothermal conditions.

Keywords: Compensation effect, activation mechanism, isokinetic temperature, vibrational energy

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### THE MENISCUS EQUATION FOR FIBER GROWTH CRYSTALS WITH A DIAMETER SMALLER THAN THE CAPILLARY CONSTANT IN THE TPS PROCESS

### Floricica BARVINSCHI, Constantin BOTA, Bogdan CĂRUNTU

**Abstract:** In this paper an analysis of the statical isotherm case of the meniscus equation for the growth of crystal fiber is proposed for the case when the capillarity conditions prevail. Using a modified Young-Laplace equation, conditions for the reachability of the growth angle are given, and the theoretical results are illustrated by numerical examples.

Keywords: meniscus equation, crystal fiber, growth angle.

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### ON A SIMPLE MODEL OF THE TYPICAL CELL FROM A SOLAR PANEL

### Minerva CRISTEA, Ioan DAMIAN, Viorel CHIRIȚOIU, Ioan ZAHARIE, Marius COSTACHE

**Abstract.** This paper presents theoretical and experimental researches concerning of application a mathematical model for a photovoltaic panel. The panel has in his structure 36 Si cells with a diameter of 5.08 cm in a serial circuit of SM1236/2 type, produced in Romania. The experimental characteristic I-U for the panel is given by the producer for different three insolations and temperatures. In order to know the behaviour of the panel and because of variation of insolation in a whole day is necessary an interpolation process who can approximate a panel which is made by an association of cells. Our model allows establishing the characteristic I-U for an average (typical) cell from serial circuit of the panel. The experimental characteristics of individual cells present a deviation from the modeled cell, but when we place the cells in a panel we obtain a statistical equalization. The simple mathematical model proposed in this paper describes the typically cell for our panel working at a constant temperature. This model allow us to determine the optimally working point for a typically cell and for a panel made by association of cells for different insolations. The results which we have obtained yield to a numerically simulation of photovoltaic panels made by one type cells in different naturally insolation conditions.

*Key words:* p-n junction, solar cell, current – voltage curves, maximum electrical power output, PV module.

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### **BALLISTIC DIODE SIMULATION VIA QHD MODEL**

#### Eugenia TULCAN-PAULESCU, Marius PAULESCU

**Abstract:** The derivation of transport models able to depict the quantum effects and to be straightforward in computer codes is a requirement of quantum device modeling. In this paper, we propose a model for ballistic transport in heterostructure devices within Quantum Hydrodynamic. The model is numerical implemented using a finite-difference discretization and tested in the case of ballistic diode.

Keywords : Ballistic transport, Quantum Hydrodynamic, Finite difference.

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