

Plastic failure mechanisms of eccentrically loaded thin-walled cold-formed steel members

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Abstract

Thin-walled cold-formed (TWCF) steel structures are usually made of class 4 sections which are prematurely susceptible to local or distortional buckling in the elastic range. As such, local plate buckling and cross-section distortion need to be a paramount part of member design, while also keeping the design method simple enough for the engineer. Failure of these members in compression and bending is always initiated by local-global interactive buckling of plastic-elastic type, and this interaction manifests as a plastic failure mechanism. This fact can be used to develop an alternative method for determination of ultimate strength of these members which would be based on the analysis of these plastic failure mechanisms.

1. Introduction

The existing European design code for steel structures, Eurocode 1993, uses the widely accepted “Effective width” method for determination of the ultimate capacity of class 4 section members, developed by von Karman in the 1930s and extended to cold-formed steel members by Winter in the 1940s. This method gives a simplified approximation of the complex nonlinear stress state in a thin plate buckled under compression. However, time consuming iterations are necessary, and it can prove to be rather complicated for thin-walled sections that are not of the simplest kind, e.g. with stiffeners on both the web and flanges. For these reasons alternate methods for calculating the ultimate loading capacity of TWCF steel members are being developed, one of them being the Analysis of the plastic failure mechanism – the subject of this paper.

Plastic failure mechanism methodology

It is known that TWCF steel members are made of class 4 sections, which experience section walls instabilities under compression loads. An attempt to eliminate this problem was made by introducing optimisation to the cross-sections by means of edge folding and stiffeners. However, when the problem of local buckling was removed, a new issue appeared - new phenomenon, later named distortional buckling of the cross-section. It can thus be concluded that the behaviour of TWCF steel members is dominated by the cross-section stability. Moreover, buckling modes can also couple together to make everything even more complex, which will be explained in the following paragraph.

Let us examine an axially loaded slender TWCF member. When the load reaches the critical value for local buckling, several buckling modes with different wave-lengths can be simultaneously induced for the same critical load value. These modes, which are periodical, will superimpose to create a localised

mode (**Error! Reference source not found.**). The member will now show unstable post-critical behaviour because the post-buckling curve of the localised mode is steeper than for periodical modes (Dubina & Ungureanu, 2000). This is the *first form of interaction* of buckling modes.

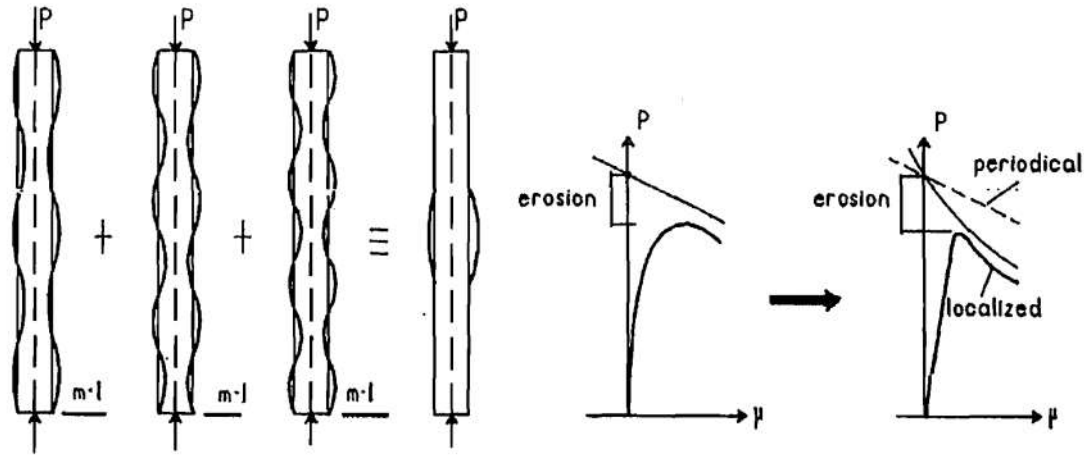


Figure 1. Interaction of local buckling modes (Dubina & Ungureanu, 2000)

The *second form of interaction* takes place when global buckling happens. Now the global buckling mode (flexural, torsional, and torsional-flexural) interacts with the local buckling mode, creating a very unstable post-critical behaviour (Dubina & Ungureanu, 2000). Due to this local-global interaction, the failure of TWCF slender members will always manifest as a plastic mechanism failure.

Several factors influence the type of plastic mechanism formed: yield strength of steel, type of the cross-section, slenderness of the member (b/t), and also the magnitude as well as the shape of initial geometrical imperfections (Hiriyur & Schafer, 2004). Additionally, including residual stresses into the analysis can complicate things even further, as showed by Schafer and Pekoz (Schafer & Pekoz, 1998). All this adds up to the fact that determining *a priori* the plastic mechanism that will form is a complex issue. On the other hand, it is necessary to correctly predict the mechanism, for the results are sensitive to the selected mechanism. Another important thing to note is that determining the load capacity for combined loading cases (i.e. compression together with bending) of TWCF members is not a linear superposition of the basic loading cases, and research on this matter is still being performed.

Yield-line analysis – brief introduction

The plastic failure mechanism is comprised of theoretically zero-width lines of yielded material, called yield lines. The analysis of yield lines is already familiar for transversely loaded concrete slabs, so a similar principle might be applicable to steel elements (**Error! Reference source not found.**). Having that idea in mind, some key differences between the two must be noted. The differences are: (1) loading is now in-plane instead of out-of-plane, (2) there is no certain way of predicting the failure mechanism that will take place in reality because the mechanism that will trigger is not necessarily the one with the lowest loading capacity (Hiriyur & Schafer, 2004). That being said, we can now differentiate two types of yield-line analyses – the classical yield-line analysis (e.g. used for concrete slabs), and the generalised yield-line analysis used for TWCF steel members. Another important difference is that the generalised yield-line analysis gives not only the loading capacity but also the load-displacement curve, which is useful for predicting member ductility, absorbed energy, etc.

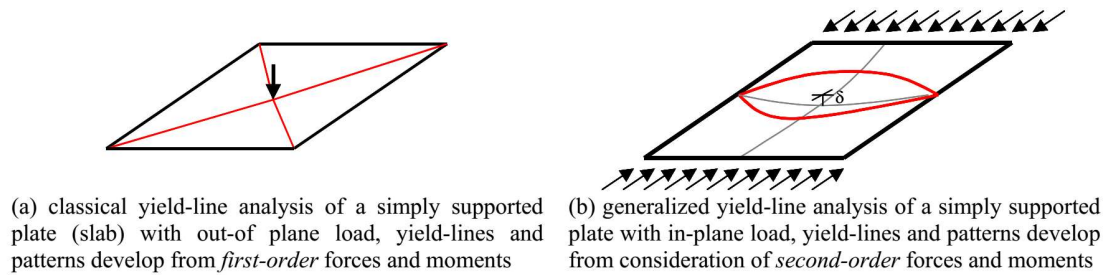


Figure 2. Typical examples of classical and generalized yield-line analysis (Hiriyur & Schafer, 2004)

2. State of art review

Since the 1960s various researchers examined TWCF plates and elements under uniform or non-uniform in-plane compression, as well as under pure bending. A conclusion from the results of these numerous experiments was that there exists a number of simple mechanisms that can be recognised. Eight types of mechanisms were distinguished for plates with mixed boundary conditions under non-uniform compression, and named *basic mechanisms* by Murray and Khoo (Murray & Khoo, 1981). In case of plates with symmetrical boundary conditions we can observe the “pitched roof” mechanism and its modifications “pyramid” and “roof” mechanism (**Error! Reference source not found.**).

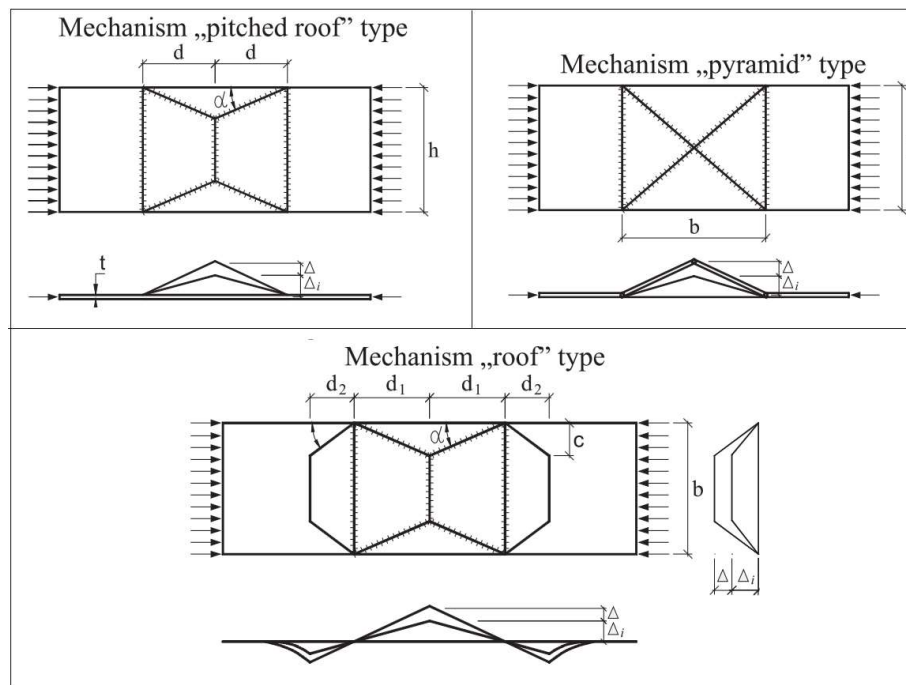


Figure 3. Basic mechanism for plates with uniform boundary conditions (Ungureanu, et al., 2010)

Relations for the load-displacement curves of all mechanisms mentioned above can be found in a comprehensive database by Ungureanu (Ungureanu, et al., 2010).

Elements under pure compression

In experiments on short plane channel-section members under compression, performed by Murray and Khoo, it was concluded that because of the asymmetry of the cross-section two groups of mechanisms can occur – named CW and CF. The CW group corresponds to the case of web in compression, and the

plastic mechanism is consequentially localised in the web; the CF group is for the flanges in compression, with the mechanism localised in the flanges. Together they total to 5 different mechanisms, presented in the aforementioned authors' paper (Murray & Khoo, 1981).

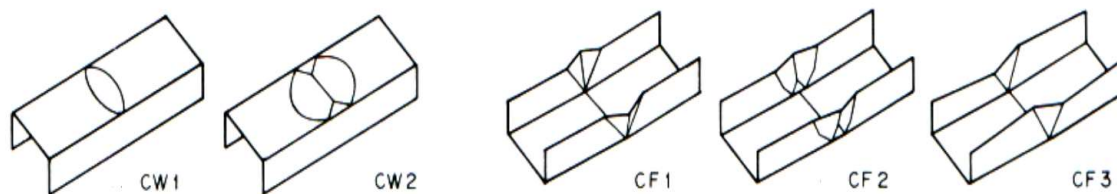


Figure 4. Basic mechanisms for channel-section columns

The principles for deriving relations for load-displacement curves for mechanisms CW1 and CF1 are explained in Ungureanu (Ungureanu, 2003).

3. Numerical (FE) analysis

The subject of this analysis is a TWCF steel C section, also called lipped-channel section, loaded with an eccentric compressive force. The force eccentricity is varied along the major axis of the section. The chosen cross-section was modelled in Abaqus software for Finite Element Analysis in order to investigate the deformed shapes, i.e. the failure mechanisms for different eccentricities.

Material properties:

- Steel S355
- $E = 210\,000 \frac{N}{mm^2}$
- $\nu = 0.3$

Section dimensions:

- $h = 150mm$
- $b = 50mm$
- $c = 15mm$
- $t = 1.5mm$
- Column length: $L = 450mm$

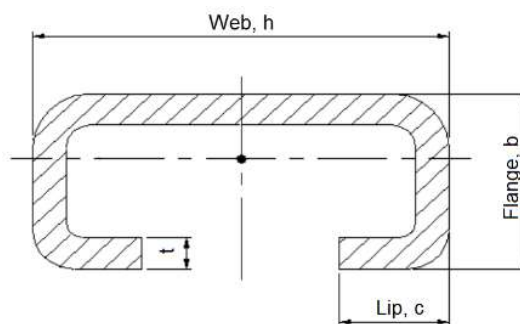


Figure 5. Lipped-channel section dimensions

The boundary conditions used were simple-simple – on one end fixed, and on the other end rolling. Torsion was restrained on both ends. The finite elements used to generate the FE mesh were 4 node shell elements with reduced integration (“S4R”) of 5mm approximate size. Local imperfections were introduced to the perfect geometry using the 3rd eigenmode obtained from a linear buckling analysis (**Error! Reference source not found.**), and scaled so that the maximum imperfection value matches the element thickness of 1.5mm.

The same model was loaded centrally as well as eccentrically along the major axis, with various positive and negative eccentricities. Riks analysis was now used to obtain the plastic failure mechanisms and the corresponding Load-Displacement curves. A displacement of 1 unit was given in the longitudinal direction at the point of interest (that point being the centre of gravity, or, later on, a point that is eccentric to the centre of gravity). Considered eccentricities were: $e = -20mm, -15mm, -10mm, -5mm, 0mm, +5mm, +10mm, +15mm, +20mm, +25mm, +30mm, +35mm, +40mm, +45mm, +50mm$.

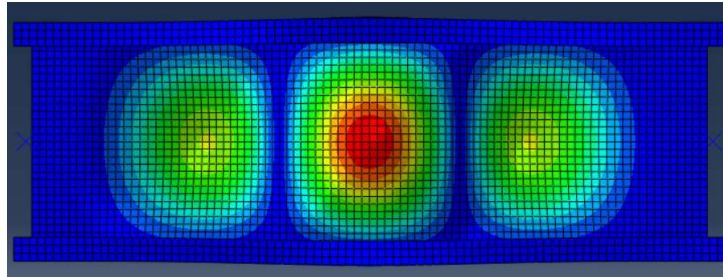


Figure 6. Eigenmode used to introduce the initial geometric imperfections

The deformed shapes obtained from Abaqus will be later used in the analytical analysis to define the failure mechanism model and its geometry. Load-displacement curves obtained numerically will be compared to the ones calculated analytically. It can be observed that the shape of the mechanism depends on the eccentricity – all negative eccentricities develop almost the same shape of mechanism (Figure 7); this is also true for positive eccentricities (Figure 8).

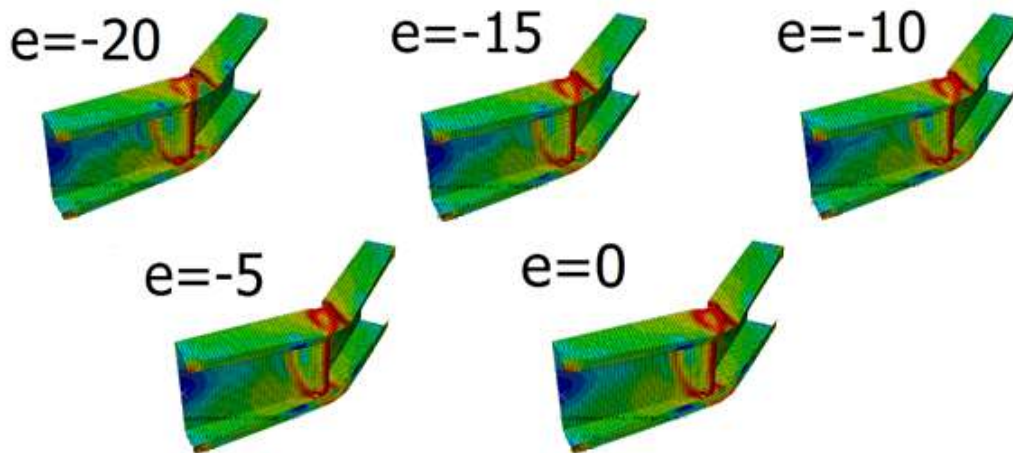


Figure 7. Failure mechanism for negative eccentricities

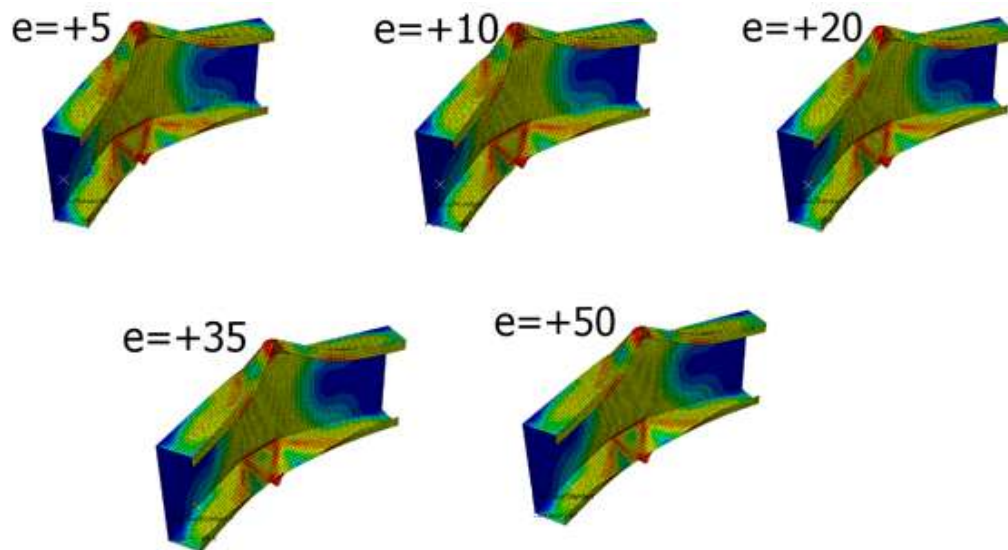


Figure 8. Failure mechanisms for positive eccentricities

4. Analytical approach – yield line analysis

Proposed failure mechanism model

The failure mechanism observed for positive eccentricities in the previous chapter was analysed using the generalised yield line analysis. It can be thought of as an enhanced CF2 mechanism from **Error! Reference source not found.** The proposed model for this collapse mechanism is shown in Figure 9. The flanges and lips of the element are in compression, and the web is in tension – hence the collapse mechanism is localised in the flanges and lips. Yield lines in lips of the section were not accounted for.

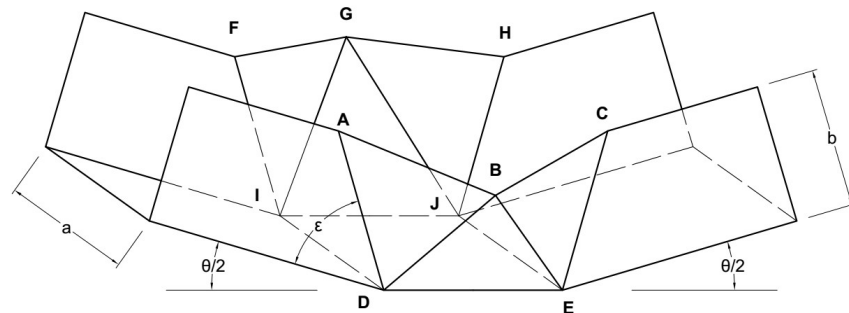


Figure 9. Proposed mechanism geometry

All walls are considered inextensible and incompressible, so all deformation happens along the yield lines only. Quantities necessary to describe deformation of the mechanism will now follow.

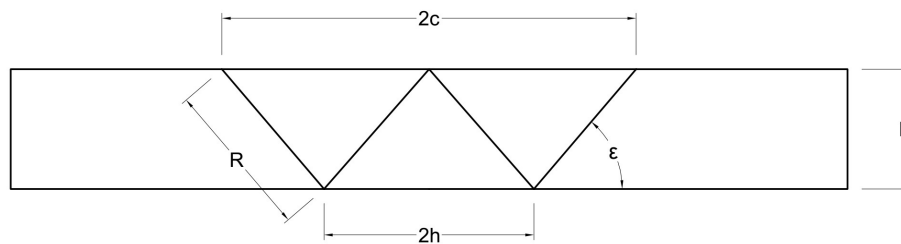


Figure 10. Side view

Quantities h and c (Figure 10) were taken from results of previous researchers (Park & Lee, 1996):

$$c = 0.87b \quad (1)$$

$$h = 0.22b \quad (2)$$

Other quantities needed are angles of relative rotation ξ , η and χ . Derivation of these angles is rather complicated, moreover, final expressions are long and complex. Therefore only the principle of obtaining these angles will be explained here.

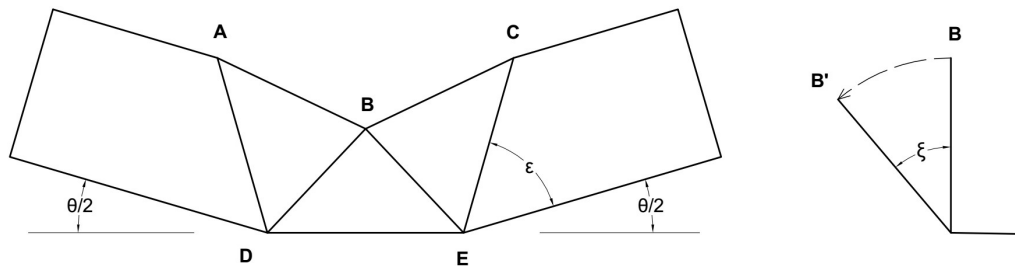


Figure 11. Deformed shape

Angle ξ is the angle of out of plane rotation of wall EBD, obtained from the condition of inextensibility of line AB (Figure 11).

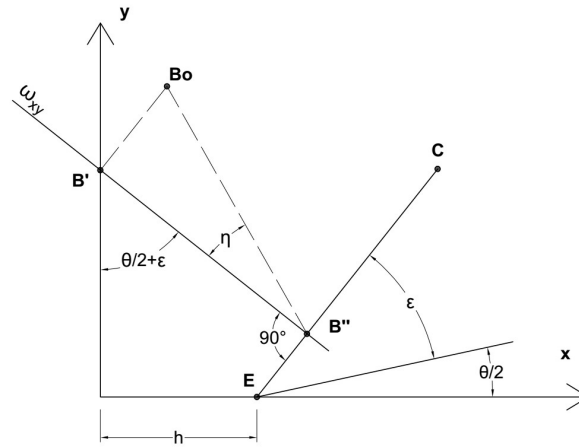


Figure 12. Angle η derivation

Angle η is the angle of relative rotation of wall ECB along the line EC. Point B' is the normal projection of point B on the vertical xy plane (plane of the flange). Plane ω , perpendicular to EC and passing through B, intersects the xy plane along the straight line ω_{xy} . Plane ω intersects EC at point B''. If we now rotate ω about line ω_{xy} so that point B falls into the point B₀ in xy plane, the angle η is the angle between lines B'B'' and B₀B''.

Angle χ is the angle of relative rotation of walls ECB and EBD. It is obtained by the same principle as angle η , with the difference that now both walls are moving in three dimensions, and because of that a normal projection of point C onto the rotating plane EBD is needed.

Analytical load-displacement curve

This paper shows results of initial yield line calculations that have been performed on the mechanism model proposed above. These results were compared to load-displacement curves obtained by means of numerical analysis.

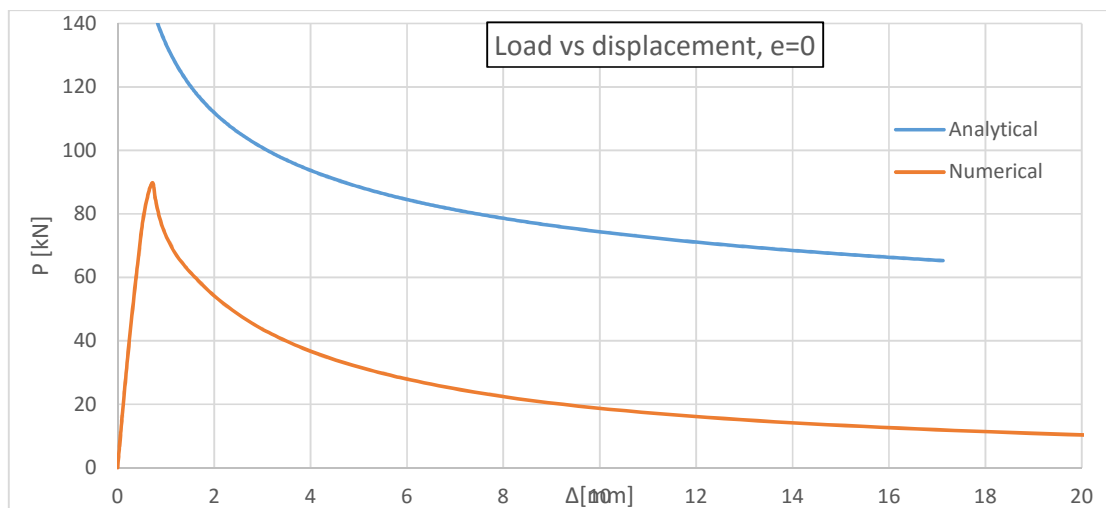


Figure 13. Numerical versus analytical curve

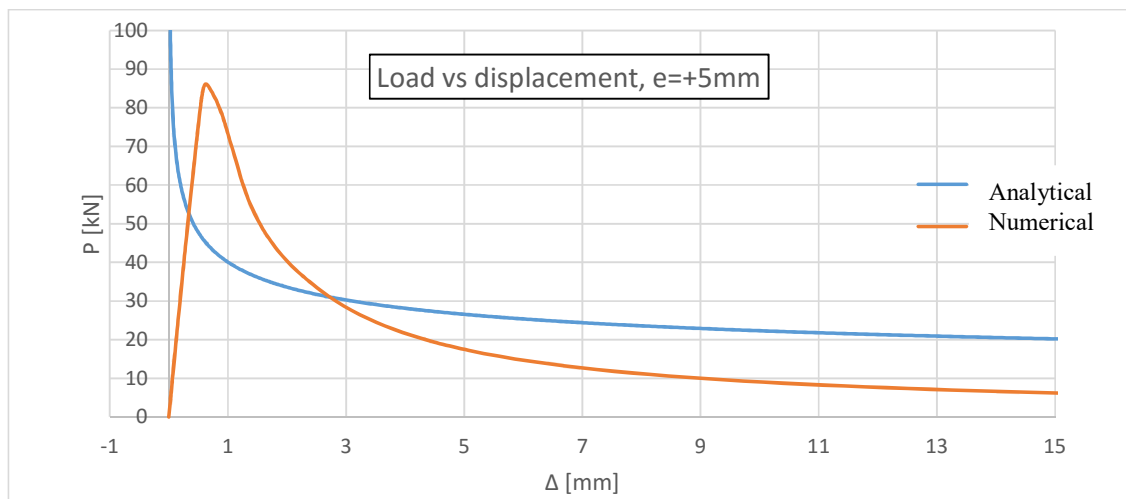


Figure 14. Numerical versus analytical curve

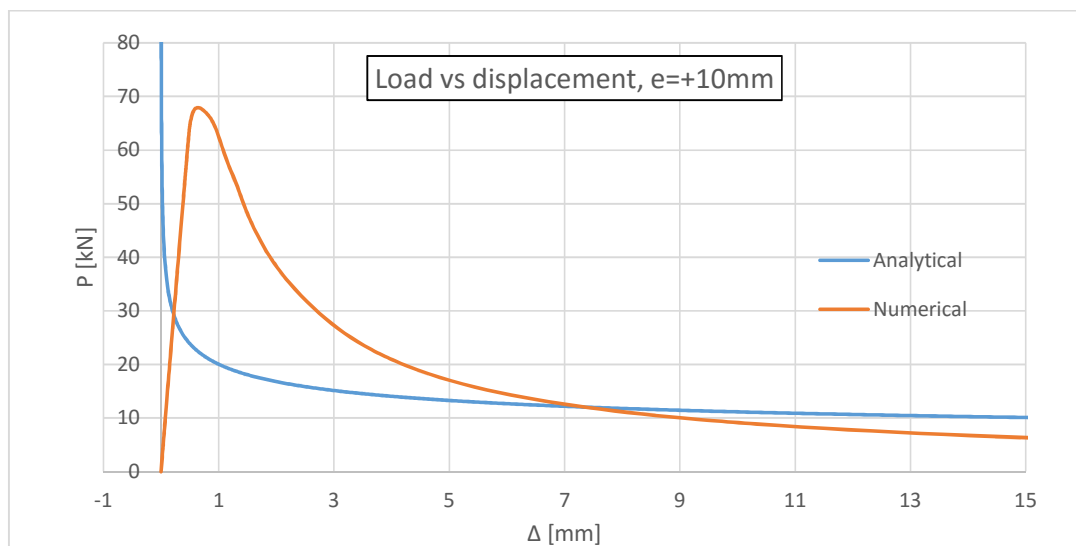


Figure 15. Numerical versus analytical curve

It can be seen that the agreement of curves is subpar. The shape of the curve seems according for smaller eccentricities (like shown above), but some translation is necessary. For larger eccentricities the agreement worsens.

5. Final remarks

The numerical analyses confirmed that short TWCF columns in eccentric compression fail by forming a plastic collapse mechanism, and therefore the yield line analysis is a legitimate approach for calculating member load capacity. Agreement of the theoretical curve with the numerical results is not very good, and the theoretical model needs more work. The final objective of this research will be to derive the analytical expression for calculating the ultimate load of an eccentrically loaded lipped-channel member. Detailed derivation of all geometric quantities and load capacity expressions will be presented in a MSc thesis of the same title.

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