

# CONTRIBUȚII PRIVIND CERCETĂRI ALE SCHIMBĂTOARELOR DE CĂLDURĂ CU SUPRAFEȚE ONDULATE CU CAPETE DREPTE ȘI SUPRAFEȚE CU GENERATORI DE TURBULENȚE

## PhD thesis – Summary

to obtain the scientific title of doctor at  
Polytechnic University of Timisoara  
in the doctorate field of Mechanical Engineering

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The thesis is structured in 7 chapters and 4 annexes. The first 3 chapters are introductory and supporting chapters in which the motivation of the thesis is explained, the current state of research in this field is presented and the concepts necessary for the rest of the thesis are introduced. The following 3 chapter describe the research work carried out on the existing extended heat transfer surfaces, but also the research upon a new extended heat transfer surface. The final chapter is reserved for the conclusions and presentation of personal contributions. The 4 annexes detail certain working procedures such as the example of error handling procedures (Anexa 1) or show the source code for the programs used in the post processing of the data (Anexa 3).

## Chapter 1 Introduction

Most of the car and equipment industry mainly uses convective mode to transfer heat between two moving fluids separated by a wall. Presently there are many constructive types of heat exchangers from simple pipes that cools a fluid, to compact heat exchangers all manufactured using different materials such as aluminum, copper, stainless steel, but also nonmetallic materials. To enhance the heat transfer, most of these heat exchangers are using extended heat transfer surfaces having different geometries. From all the extended heat transfer surfaces types the most used are the continuous fins, wavy fins, louvered fins, and the offset fins. Each one of this kind of has advantages and disadvantages which makes them useful for different fluids and areas of operations. Because the performances of this kind of extended heat transfer surfaces is influenced by a variety of factors (i.e. the flowing speed, the temperature, the working fluid and the shape and the geometry dimensions) their research is mainly experimental and numerical. For this reason, designing and dimensioning the heat exchangers requires intensive consultation for compendia such as Kays and London [1]; or information from literature such as Wang et al. [2], Dong et. al. [3]–[5] that contain experimental information about the criterial equations of different extended heat transfer surfaces. The problem of these criterial equations is that is applicable only on the experimental domain and the geometry for which they were raised having with differences of  $\pm 30\%$ . These differences, which can be attributed to variations in geometry between manufacturers, make the design and sizing of heat exchangers rather cumbersome and often inaccurate.

As the surplus heat in industrial processes must finally be discharged into the atmosphere, most of the heat exchangers used are air-cooled heat exchangers. In these heat exchangers the air's convection coefficient determinates the heat performance because it is the lowest one (10 – 200

[W/m<sup>2</sup>K]) regarding the other fluids as water (~3000 [W/m<sup>2</sup>K]) or lubricants (500 – 1000 [W/m<sup>2</sup>K]). For this reason, most of the research on the extended heat transfer surfaces is directed toward increasing the performance of cooling air.

The reason for this work is the RAAL's need to grow and develop, which is the main manufacturer of aluminum heat exchangers from Romania. The objectives of this thesis are represented by the company's need to optimize and enhance its dimensioning software and develop better products which can be stated as follows:

- Experimental determination of criterial relations regarding the heat and hydraulic performances of extended heat transfer surfaces
- Comparison of different performance criteria regarding the extended heat transfer surfaces
- Numerical simulation of the 3D models of such surfaces with the experimental validation with the aim at reducing the experimental tests.
- Designing new types of geometry for improving the overall performances of extended heat transfer surfaces.

To achieve the objectives described above and taking into account what has already been said: The importance of the air thermal transfer coefficient, as well as the deviations between the results of the various research; this work presents experimental determination of the criteria equations for the family of *wavy fins with straight ends* the most commonly used type of extended heat exchange surface for air from the RAAL production, and the numerical analysis together with experimental model validation for this fin. Finally, are showed the comparative numerical studies between the *wavy fins with straight end* and the *wavy fin having rectangular cutouts*, a new kind of geometry, done by inserting rectangular cutouts to induce secondary airflow to improve the mixture of the bulk flow

## Chapter 2. Extended heat transfer surfaces

In this chapter the most important extended heat transfer surfaces used in the compact heat exchangers are reviewed:

- Offset fins a.
- Louvered fin b.
- Longitudinal turbulence generators c.
- Wavy fin d.

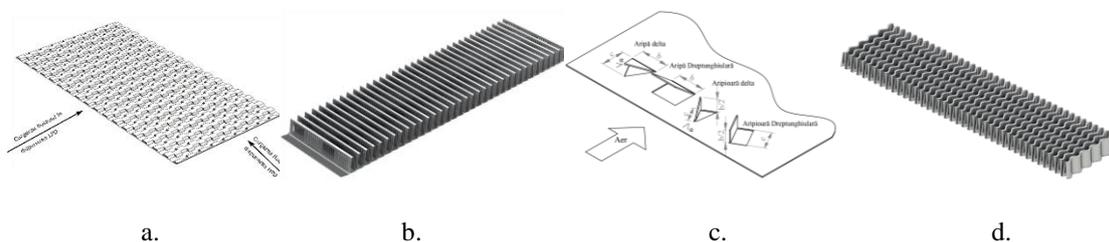


Fig. 1 types of extended heat surfaces

For the first three extended heat transfer types (a, b, c, from figure 1) the description is briefly described the way in which these extended surfaces achieve the increase of the heat exchange coefficient, the preferred fluid for which they are used: gas or liquid, or the state of research but also it is described the advantages and disadvantages in pressure drop, to remember here is the paper [6] in which the performances of the offset fin is analyzed.

The wavy fin (figure 1.d), together with its variations, it is the most used geometry for extending

the heat transfer surface for the air. Increasing the heat exchange is done by breaking the boundary layer when the air goes over the corrugations, and the mixing is done with the help of vortexes with the rotation axis being perpendicular on the flow direction. The advantage of this kind of geometry is given by the high efficiency of the heat transfer and for the fact that they have a great resistance to clogging [7] which makes it good for the dusty environments. Since this kind of extended heat transfer surface is the most used in production it is also the most studied. In the literature can be found many papers, most of them dealing with some aspect or other of this kind of extended surface. The most representative papers, the papers that describe experimental criteria relations, are Ismail [8], Dong with its three papers [3]–[5], and Aliabadi [9]. Using the specific geometric parameters in the RAAL production, a comparison of these 4 criteria relationships has been carried out.

The result of the comparison of these relations shows us a notable difference for some of them. Thus, the estimation made with Dong’s work [5] it is very different from the estimation done using the other relations, the difference is between 30% and 85% as can be seen from figure 2 for the heat transfer and figure 3 for the friction coefficient

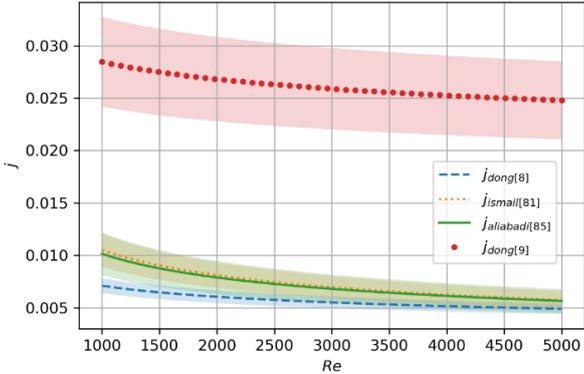


Fig. 2 Colburn number with error bands for the different relation taken from literature

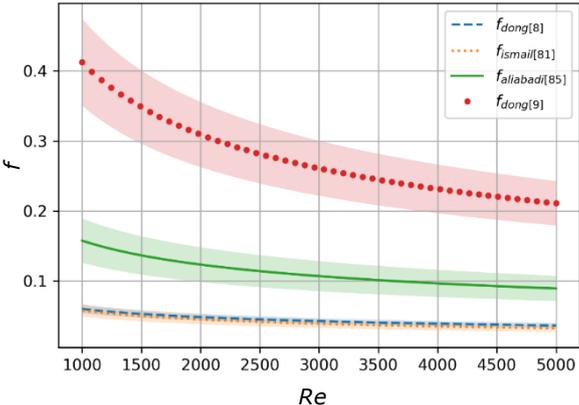


Fig. 3 Fanning friction coefficient of the literature relations

The results from the other authors for the heat exchange are relatively close, their difference is inside the error interval as it was declared by the authors (between 3 and 30%). But for the friction coefficient the results are completely different, besides Dong [4] and Ismail [8] that have a 5% difference the rest have differences between 60 and 85% which makes them totally incomparable

These differences between the literature relations makes them unusable for sizing the heat

exchangers within RAAL.

This chapter closes with the presentation of three efficiency criteria usually used in comparing extended heat transfer surfaces. These are the area goodness factor  $j/f$ , volumetric goodness factor  $j/f^{1/3}$  and the JF factor which compares based on a reference fin.

### Chapter 3 Heat Exchanger Types

This chapter is a support chapter in which three methods of classification of heat exchangers are presented: after the phases of the working fluids, after the flow mode of the primary fluid regarding the secondary fluid and upon their construction. The chapter presents in detail two methods for calculating the heat exchangers. The first one is the *logarithmic temperature difference* which is especially applied to coolers having counter / echi current circulation, or if one of the fluids suffers a phase change. This method is used for postprocessing the simulation data. The second method is the so-called  $\epsilon$ -NTU. This method can be applied to crossflow circulations, this method being used for postprocessing the experimental data.

### Chapter 4. Experimental study of wavy fin with straight ends.

Because the literature data have a great spread and can't be used in RAAL's dimensioning software, but because the wavy fin produced in RAAL has a modified geometry regarding the ones from the literature – the wavy fin has straight sections at both ends as seen in figure 4 [10]–[12] -, it was necessary to conduct experimental test with the purpose of finding criterial relations to describe the heat and hydraulic performances

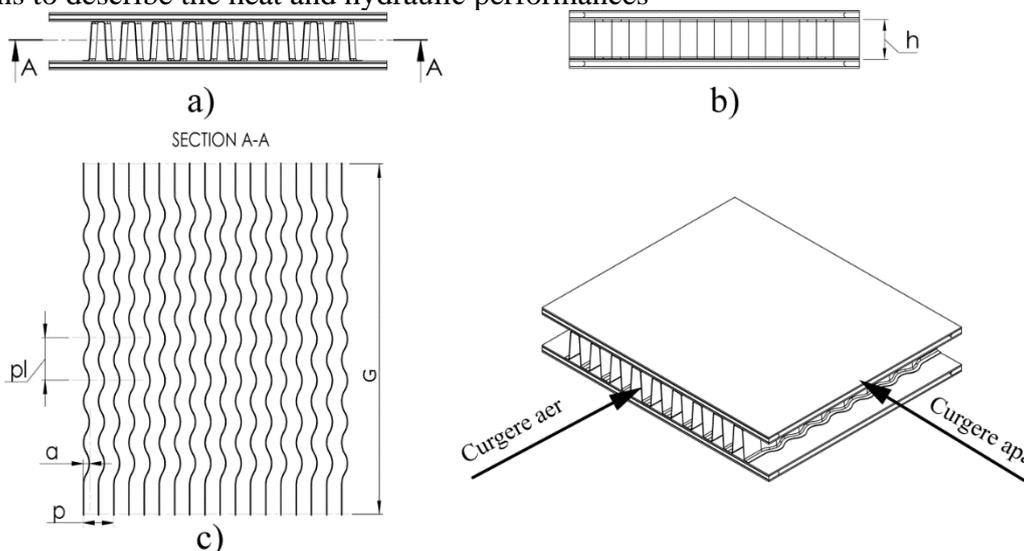


Fig. 4 Air structure of the test heat exchangers: a) frontal section, b) lateral section, c) fin section

The form of the relations to be found is the one from the equations (4.1).

$$Nu = n \cdot Re^b \cdot Pr^c \prod_{i=h,p,g} F_i^{d_i} \quad (4.1)$$

$$f = e \cdot Re^j \prod_{i=h,p,g} F_i^{k_i}$$

were  $n, b, c, d_i, e, j, k_i$  are the parameters which must be experimentally determined and  $h$  (height),  $p$  (pitch),  $g$  (thickness) and  $l$  (length) are the geometric parameters of the wavy fin,  $F_i = i/D_h$  the nondimensional factors of the equations

The experimental tests were done on 17 test heat exchangers (figure 5) having different air structure using the RAAL's test bench for which the functional sketch is presented in figure 6.

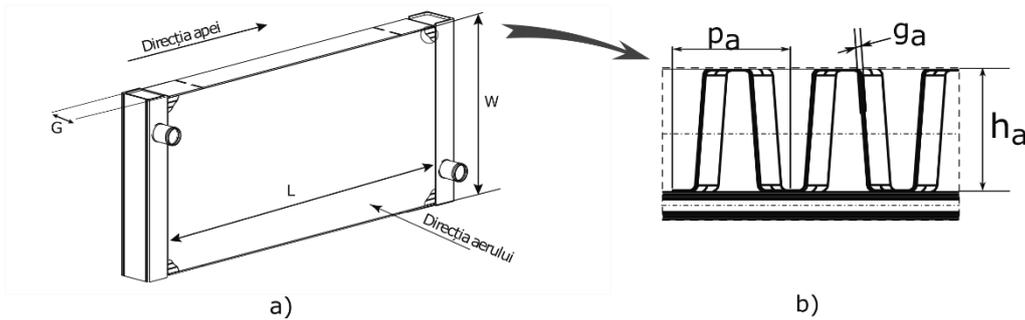


Fig. 5 Water coolers sketch  
a) the complete heat exchanger b) wavy fin detail

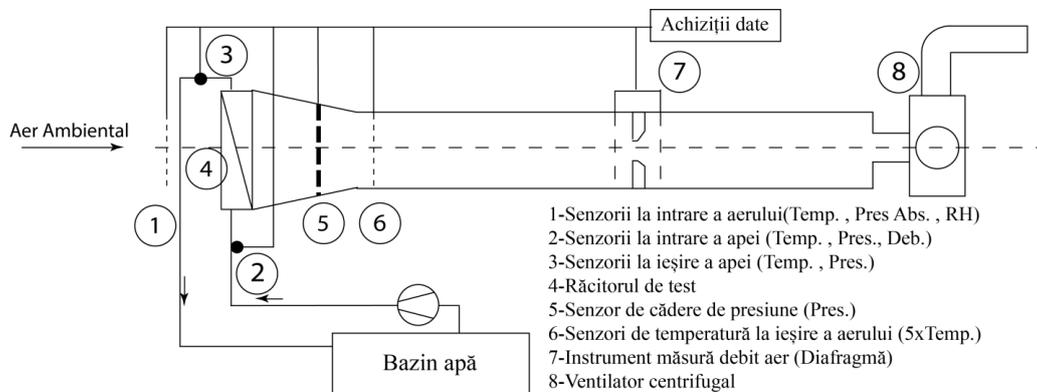


Fig. 6 test bench functional sketch

Because the measurements had to be done in a steady state manner for each functioning point the measurements were repeated between 5 to 10 times in a time span of 2 min. The measurements obtained in this way permitted to calculate the uncertainties intervals as in [13] detailed in Anexa 1.

On each heat exchanger has been measured in 30 to 50 operating points, reaching more than 760 measuring points on all coolers.

The experimental data obtained, presented in the figures 7 and 8, was postprocessed using the  $\epsilon$ -NTU method implemented using python with the help of the scientific packages such as numpy, scipy, matplotlib and pandas (with source codes are presented in Anexa 3). For the calculation of the water convection coefficient I used the literature expressions [14] and [15].

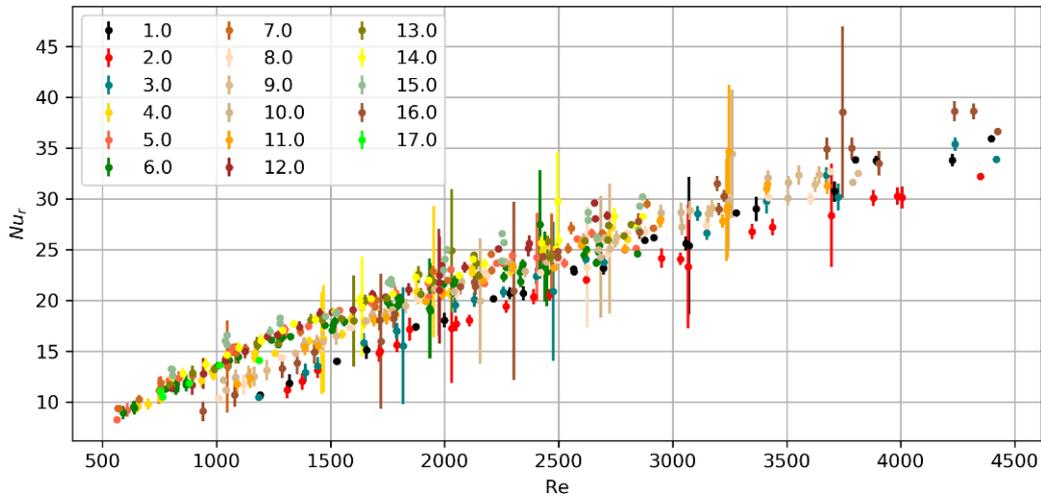


Fig. 7 Nusselt number as a function of Reynolds number for all the coolers tested

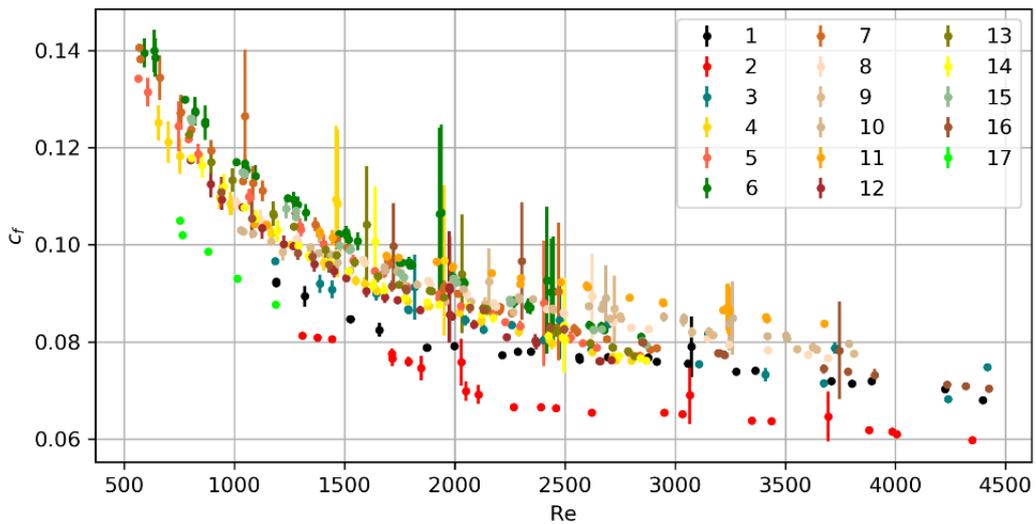


Fig. 8 Friction coefficient as a function of Reynolds number for all the coolers tested

Next, using the property of the expressions in (4.1) to be linearized (by applying the natural logarithm) I was able to find the coefficients of the above functions by means of a linear regression and I have obtained the expressions of the form (4.2) and (4.3) below

$$Nu_r = 0.848 \cdot Re^{0.75 \pm 0.014} \cdot F_p^{-3.27 \pm 0.19} \cdot F_h^{-0.55 \pm 0.04} \cdot F_g^{0.31 \pm 0.022} \quad (4.2)$$

$$c_f = 1.0 \cdot Re^{-0.28 \pm 0.007} \cdot F_p^{-1.34 \pm 0.10} \cdot F_h^{-0.18 \pm 0.022} \cdot F_g^{-0.048 \pm 0.012} \quad (4.3)$$

The expressions (4.2) and (4.3) approximate 95% of the experimental points within  $\pm 15\%$ , with a mean deviation of 2.3% and respectively 0.6%.

The chapter concluded with a study using the effectiveness relations that shows the presence of a maximum performance zones for this kind of extended heat transfer surface.

## Chapter 5. Numerical study of way fin with straight ends

The experimental testing can become very costly if we want to study geometric parameters such as the amplitude of the corrugation or the length of the straight section, because modification of these parameters requires a redesigning of the tooling with which the fin is produced. In this case the best mode to study the influence of these kind of parameters is by simulation using CFD codes

This chapter begins by reviewing the different methods of discretization and of the turbulence models used in this simulation software and continues with a brief description of how they are implemented in the SolidWorks simulation system. The SolidWorks system uses a discretization model based on the finite volume method (FVM) and the mesh is a structured one with the cells being cartesian based. The turbulence model is implemented as a two-equation model  $k-\epsilon$  modified. Due to the fact that RAAL's CAD system uses the SolidWorks ecosystem it was convenient to use the CFD solution from SolidWorks.

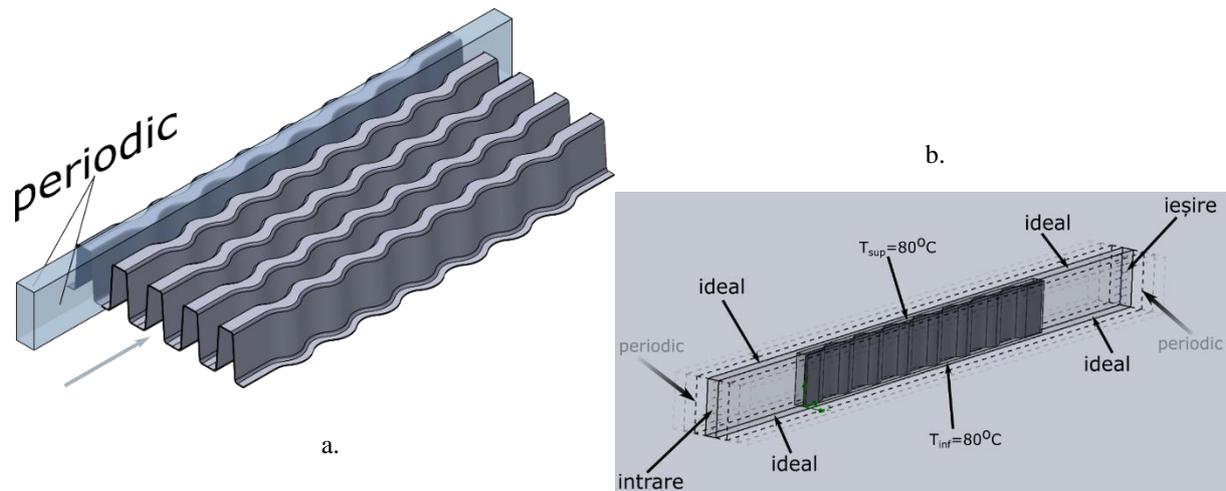


Fig. 9 a. Fin model, b. boundary conditions

Before simulating the wavy fin with straight ends, its geometry must be simplified in order to reduce as much as possible the mesh size. Thus from figure 9.a it can be seen that the wavy fin has a repetitive geometry which can be simplified by removing the elements that are repeated. In this figure 9.b is represented the simulation domain along with the boundary conditions imposed

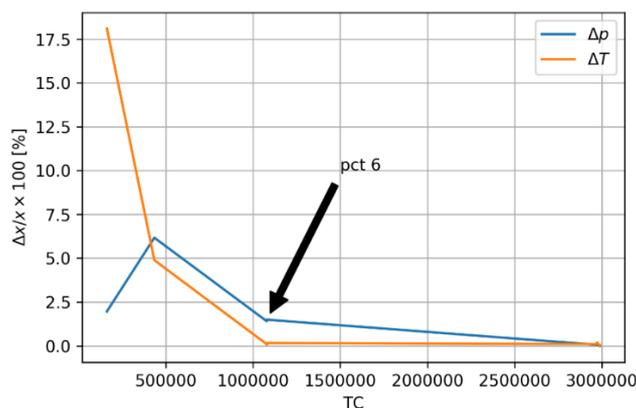


Fig. 10 Relative difference of  $\Delta T$  and  $\Delta p$  as a function of mesh size

Choosing the mesh level is a compromise between the maximum acceptable errors, the time it takes to simulate the results and the hardware available. In practice the mesh level it is chosen by the rule that states that when doubling the mesh size, the results of interest doesn't modify significantly (<5%) then the results can be considered mesh independent. In our case the mesh

independence for  $\Delta T$  and  $\Delta p$  is achieved at a mesh size greater than  $10^6$ .

In order to be able to make a comparison of the results obtained from simulation with the experimental ones we must postprocess the first and bring them in a dimensionless form by expressing the heat exchange as a Nusselt number and the pressure drop as a friction coefficient. This can be done by using the logarithmic temperature difference, from chapter 3, were the wall temperature is maintained at a constant level.

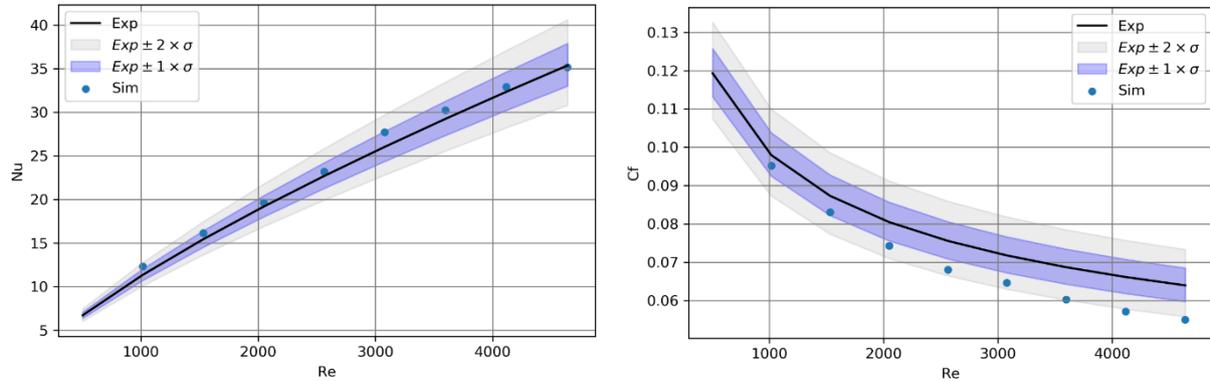


Fig. 11 Comparison of the simulation and experimental results

The comparison results are presented in figure 11. In this figure the simulation results are marked as dots and the experimental results are represented by the solid line. Analyzing this figure reveals that the simulation approximates very well the experimental results. For the simulated Nusselt number, it has been obtained a mean deviation of 6.7% from the experimental results and for the friction coefficient it has been obtained a 9.4%.

In conclusion we can say that this numeric model is valid and can be used in further studies.

## Chapter 6 Numerical study of wavy fin with rectangular cutouts

The process that the wavy fin with straight ends used to intensify the heat exchanged is by creating vortices with the rotation axes perpendicular on the flow direction (figure 12a.). The mixing process has the effect of bringing cooler air near the fin wall, which results in a higher temperature gradient, thus increasing the convection coefficient.

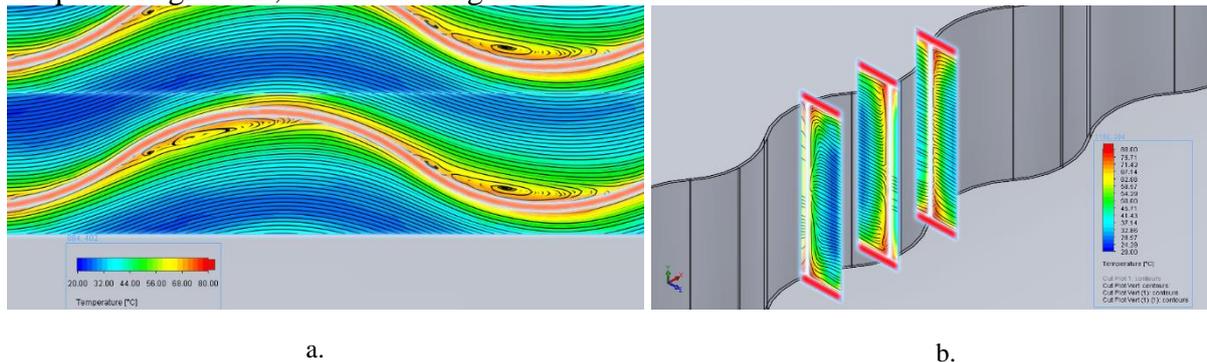


Fig. 12 Speed variation of a wavy fin with straight ends: a. in flow direction b. perpendicular on flow direction

Increasing the mixing and implicitly increasing the convection coefficient can be achieved by increasing the amplitude of the corrugations but as it was showed in [16] this modification comes with a high pressure drop increase.

Analyzing the speed profile on the  $yz$  plane – the plane perpendicular on the flow direction ( $x$  axis) – revealed that there is no mixing on this plane (figure 12b.) thus the colder air regions from the bulk flow doesn't mix with the warmer zones near the wall, which lowers the

convection coefficient.

For further improvement of the convection coefficient, I started from the idea of longitudinal turbulence generators (WG) presented in the chapter 2, to induce a secondary flow in the yz plane to further mix the air through the fin

The wavy fin (AOcd) modification must meet the criteria listed below:

- Be able to generate a helical movement of the air through the fin which will further limit the increase of the boundary layer
- To keep the field advantage of the wavy fin by having a high resistance to clogging
- Its manufacturing has to have a low technological effort

To comply with the above conditions, the fin (AOcd) has been modified by applying rectangular cutoffs in an offset pattern as in figure 13 below

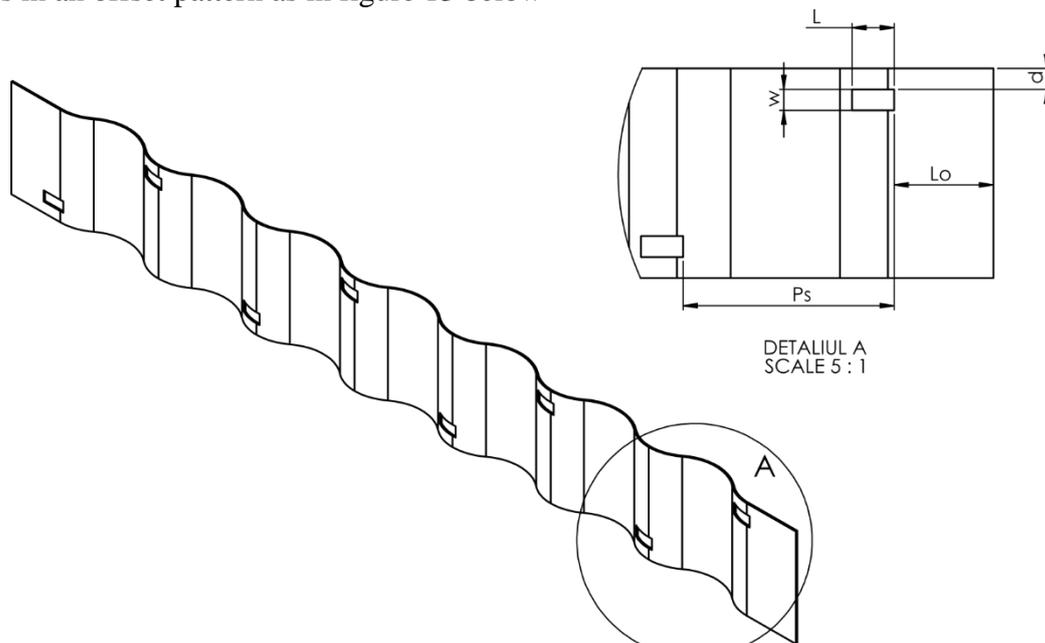


Fig. 13 Wavy fin with rectangular cutoffs- AOsr

To completely describe the new geometry (figure 13) I have introduced 5 new geometrical parameters which describe the position ( $d$ ,  $L_0$ ), dimension ( $L$ ,  $W$ ) and the number of cutoffs ( $P_s$ ). Because the performance influence of the other parameters such as height, pitch and thickness are already known these ones will be removed from this study and will be considered constants having the values of the ones in the previous chapter.

Finding the optimal values for the parameters was done using the DoE (design of experiment) method. Considering the parameters with only 2 levels I have conceived a full factorial experiment with 32 simulation, and I have used the JF as the objective function. For these simulations the speed was considered constant at 5 m/s. The results were centralized, and all the geometric parameters were normalized in the  $[-1, 1]$  interval. The study of the effect of each geometric parameter on the JF objective function was done using the methods described in [17] and presented in figure 14 below:

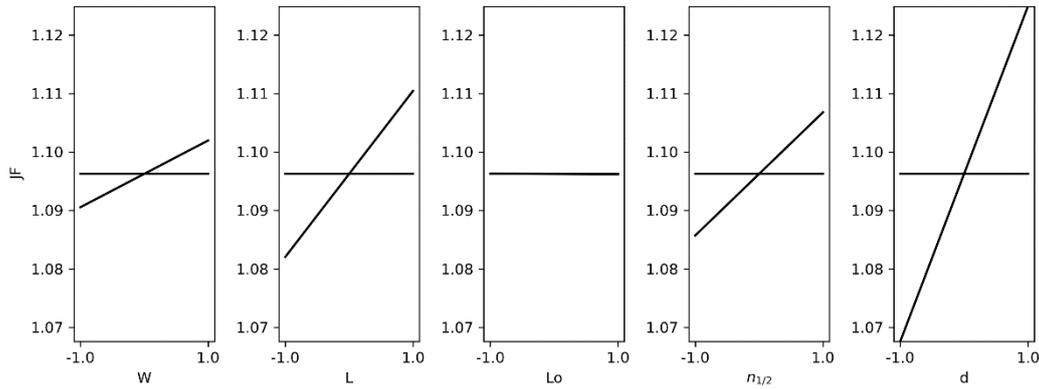


Fig. 14 Parameters effect on JF objective function

From the figure 14 it can be observed that the horizontal position parameter  $L_0$  doesn't have any effect on the JF and the vertical position parameter  $d$  has the biggest influence. Next, I have studied the effect of the interaction of these parameters by describing the JF function as a linear function of the parameters and their interactions as in equation (6.1) which allowed me to use the StatsModels [18] package for a linear regression. The linear regression revealed that the interaction between the width  $W$  and the vertical position  $d$  of the cutoff ( $C_{Wd}$ ) has a great negative influence in the objective function

$$JF = A + \sum_i B_i \cdot X_i + \sum_{j=i+1} C_{ij} \cdot X_i \cdot X_j + \sum_{j=i+1} \sum_{k=j+1} D_{ijk} \cdot X_i \cdot X_j \cdot X_k + E \cdot X_1 \cdot X_2 \cdot X_3 \cdot X_4 \quad (6.1)$$

were  $A$ ,  $B_i$ ,  $C_{ij}$ ,  $D_{ijk}$ ,  $E$  are the regression coefficients,  $X_i$  one of the parameters with:  $1 \rightarrow W$ ,  $2 \rightarrow L$ ,  $3 \rightarrow n_{1/2}$ ,  $4 \rightarrow d$ .

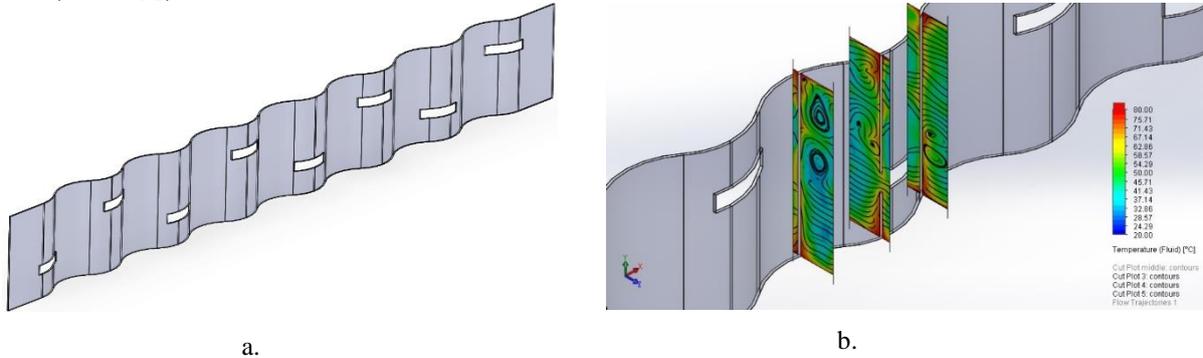


Fig. 15 a. Optimum geometry of the way fin with cutoffs; b. speed variation on a plane perpendicular to the flow direction

**Table 1 Optimum parameter configuration**

W [mm]	L [mm]	$L_0$ [mm]	$n_{1/2}$	d [mm]	JF
1	4	4.5	4	2.5	1.28

The optimum parameter configuration for which the JF function to be at a maximum was done using the numerical *minimize* function implemented in scipy which used the SLSQP algorithm [19], were the maximization problem was changed into a minimization problem by changing the sign of the JF function. The parameter configuration for which the JF function has a maximum in the studied range are presented in the table 1 with the geometric form as in figure

15a. As it can be seen in figure 15b this geometry has a greater mixing of the bulk flow than the wavy fin with straight ends.

With this geometry I have conducted simulation in the speed range of 4 – 19 m/s and the results are presented in the figure 16 as JF criteria

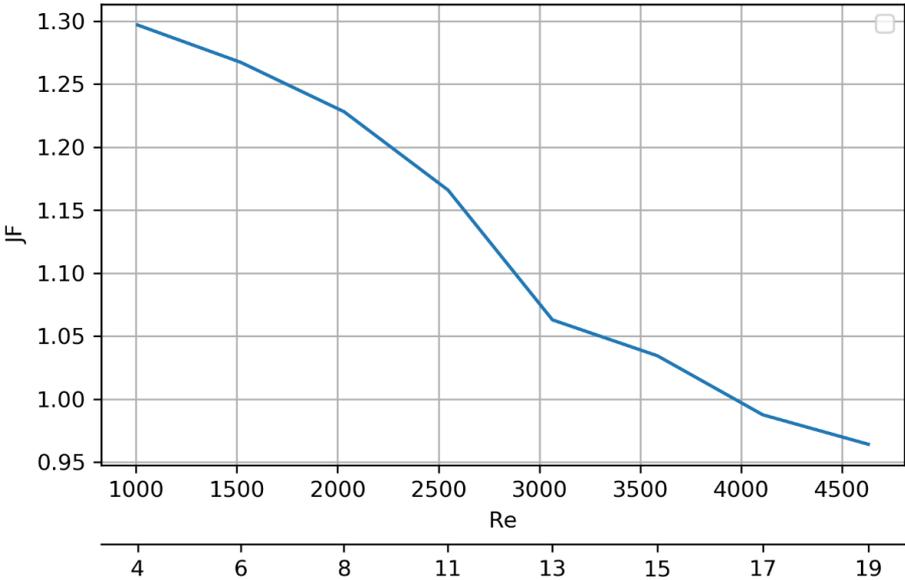


Fig. 16 JF criteria for the new geometry

In conformity with the JF criteria (figure 16) the new geometry has a net advantage for air speeds smaller than 10 m/s. According to the definition the JF criteria can be used only to compare the fins at the same Reynolds number. In real application the situation can be different. The real applications use a fan for circulating the air through the cooler which has a flow curve also (over pressure as a function of flow). There are situations in which the fan can't be modified, or his modification implies greater costs. For this case I have imagined an experiment in which I have compared the field performances of a single fin by answering to the following question: *Which are the thermal performances of a heat exchanger with the same fan and the same working conditions if we change the air fin with the new one?*

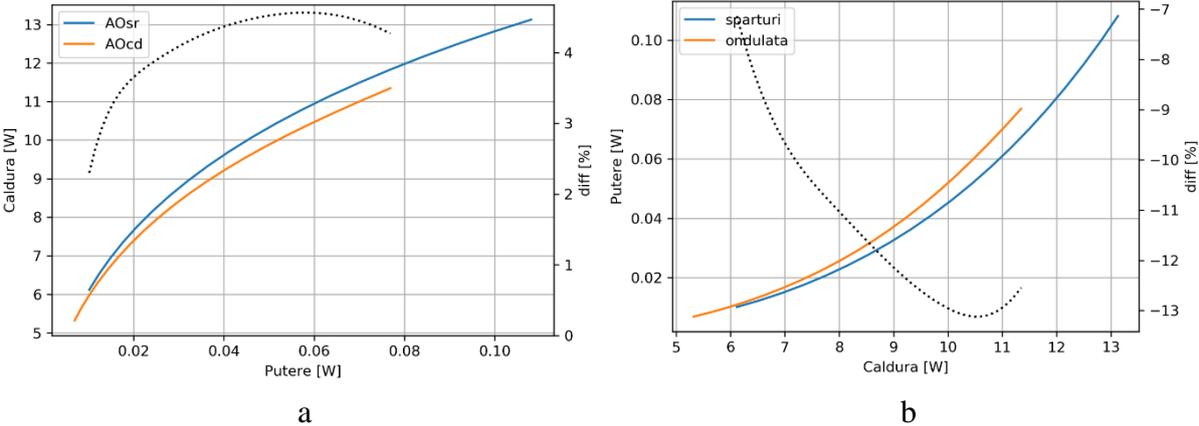


Fig. 17 a. Heat flux as function of circulation power; b. Circulation power as a function of heat flux

There are two answers for the above question: The first one is represented in the figure 17a, and it shows that in the case of the same recirculating power (different air flow) the new fin has a greater heat rejection with 2 to 4% in the studied speed range. The second answer is represented by the figure 17b, and it shows that if we want to maintain the same heat rejection then we will

need a lower power for the fan which is comprised between -7 to -13% in the studied speed range.

## Chapter 7. Conclusions and personal contributions

In the present work, theoretical and experimental researches were performed regarding the thermal and hydraulic performances of the heat exchangers with corrugated surfaces with straight ends to obtain a new geometry, which will lead to the increase of these performances. Research in the field of extended heat transfer surfaces is a very active field, having a great importance for the industry of heat exchangers.

More specific, for the RAAL company knowing the analytical expression of the Nusselt number and friction coefficient variation with the geometrical parameters for the wavy fin with straight ends is important. Therefore, in this thesis 17 test coolers – with different geometric parameters – were used to determine the coefficients of such expressions

Designing and testing the heat exchangers is a lengthy and costly process, therefore studying the new geometries firstly on the computer with the help of simulation will reduce and improve this process. The numeric results in this paper were validated with the help of the experimental results that were obtained on the wavy fin with straight ends. Here it has been shown that there is a good correlation between the experimental and simulated results within  $\pm 15\%$

The experimental and numerical researches done of the air flow through such extended heat transfer surfaces were materialized with a few personal contributions such as:

- the analysis and systematization of the information for extended heat transfer surfaces found in the literature up to the present moment. The analysis focused for the presentation of the main criterial relations for the wavy fins, and it has sowed the difficulty in obtaining a general valid criterial equation;
- the test bench has been enhanced by inserting some automatization in postprocessing data tests;
- were designed and tested 17 aluminium heat exchangers, having different geometrical parameters and dimensions, in over 760 working points;
- analysis and determination of criterial equations (4.2) and (4.3) which allow the calculation of the heat and hydraulic performance of the wavy fin with straight ends as a function of its geometrical parameters;
- the automation postprocessing of large number of data obtained on many heat exchangers using the Python's scientific ecosystem. Source code presented in A3
- improvement of the RAAL's calculation program (PCSC) with the use of the (4.2) and (4.3) relations;
- designing and analyzing the performances of a new geometry, namely *wavy fin with rectangular cutoffs*, that uses the longitudinal turbulence generators to induce a secondary flow through the fin in order to improve its overall performances

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